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MODIFIED HOMOTOPY PERTURBATION METHOD FOR SOLVING TWO-DIMENSIONAL FUZZY FREDHOLM INTEGRAL EQUATION

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Abstract: In this paper, a numerical method based on modified homotopy perturbation method is presented for solving a linear two-dimensional fuzzy Fredholm integral equation of the second kind (2D-FFIE-2). We use parametric form of fuzzy functions and convert a 2D-FFIE-2 to a linear system of Fredholm integral equations of the second kind with three variables in crisp case. We use the modified homotopy perturbation method to find the approximate solution of the converted system, which is the approximate solution for 2D-FFIE-2. The solved problems reveal that the proposed method is effective and simple, and in some cases, it gives the exact solution.

AMS Subject Classification: 45BXX, 45FXX
Key Words: two-dimensional fuzzy Fredholm integral equation, parametric form of fuzzzy integral equation, modified homotopy perturbation method

1. Introduction

The concept of integration of fuzzy functions was first introduced by Dubois and Prade (1982). Alternative approaches were later suggested by Goetschel and Voxman (1986), Kaleva (1987), Matloka (1987) and others. The topic of fuzzy

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integral equation (FIE) which growing interest for some time, in particular in relation to fuzzy control, have been rapidly developed in recent years. Wu and Ma (1990) investigated the fuzzy Fredholm integral equation for the first time.

Recently, some numerical methods have been introduced to solve linear fuzzy Fredholm integral equation of the second kind in one-dimensional space (FFIE-2) and two-dimensional space (2D-FFIE-2). For example, Effati et al. [9] and Rivaz et al. [1] use homotopy perturbation method for solving FFIE-2 and 2D-FFIE-2, respectively. In this work, we use the modified homotopy perturbation method (MHPM) for solving 2D-FFIE-2.

The remainder of this paper is organized as follows: in Section 2, we present the basic notations of fuzzy numbers, fuzzy functions and fuzzy integrals. In Section 3, the 2D-FFIE-2 and its parametric form are discused. We explain the modified homotopy perturbation method for 2D-FFIE-2 in Section 4. Then we apply the method to some examples, and compare the results with the exact solutions in Section 5. Conclusions are given in Section 6.

2. Preliminaries

Definition 1. A fuzzy number is a fuzzy set $u : R^1 \to [0, 1]$ wich satisfies:

1. u is upper semicontinuous.
2. $u(x) = 0$ outside some interval $[c, d]$.
3. There are real numbers a, b: $c \leq a \leq b \leq d$ for wich
   (a) $u(x)$ is monotonic increasing on $[c, a]$,
   (b) $u(x)$ is monotonic decreasing on $[b, d]$,
   (c) $u(x) = 1$, $a \leq x \leq b$.

Definition 2. The parametric form of a fuzzy number u is a pair of functions $(\underline{u}(r), \overline{u}(r)), 0 \leq r \leq 1$, which satisfies the following requirements:

1. $\underline{u}(r)$ is a bounded, continuous, monotonic increasing function over $[0, 1]$.

2. $\overline{u}(r)$ is a bounded, continuous, monotonic decreasing function over $[0, 1]$.

3. $\underline{u}(r) \leq \overline{u}(r), 0 \leq r \leq 1$.

$(\underline{u}(r), \overline{u}(r)), 0 \leq r \leq 1$, are called the r-cut sets of u.

The set of all fuzzy numbers is denoted by E^1.

Definition 3. For arbitrary fuzzy numbers $u = (\underline{u}(r), \overline{u}(r)), v = (\underline{v}(r), \overline{v}(r))$ and real number k, we have

$$(u+v)(r) = (\underline{u}(r) + \underline{v}(r), \overline{u}(r) + \overline{v}(r)),$$

$$(u-v)(r) = (\underline{u}(r) - \overline{v}(r), \overline{u}(r) - \underline{v}(r)),$$

$$(ku)(r) = \begin{cases} (k\underline{u}(r), k\overline{u}(r)) & k \geq 0, \\ (k\overline{u}(r), k\underline{u}(r)) & k < 0. \end{cases}$$

Lemma 4. *Let $(\underline{u}(r), \overline{u}(r))$, $0 \leq r \leq 1$, be a given family of non-empty intervals. If*

1) $(\underline{u}(r_1), \overline{u}(r_1)) \supset (\underline{u}(r_2), \overline{u}(r_2))$ for $0 \leq r_1 \leq r_2 \leq 1$,

2) $(\lim_{k\to\infty} \underline{u}(r_k), \lim_{k\to\infty} \overline{u}(r_k)) = (\underline{u}(r), \overline{u}(r))$, whenever (r_k) is a non-decreasing converging sequence converges to $r \in [0,1]$.

Then the family $(\underline{u}(r), \overline{u}(r)), 0 \leq r \leq 1$, represents the r-cut sets of a fuzzy number $u \in E^1$.

Conversely, if $(\underline{u}(r), \overline{u}(r))$, $0 \leq r \leq 1$, are the r-cut sets of a fuzzy number $u \in E^1$, then the conditions (1) and (2) hold, see [3].

Definition 5. For arbitrary fuzzy numbers $u = (\underline{u}, \overline{u})$ and $v = (\underline{v}, \overline{v})$ the quantity

$$D(u,v) = \max \left\{ \sup_{0 \leq r \leq 1} |\underline{u}(r) - \underline{v}(r)|, \sup_{0 \leq r \leq 1} |\overline{u}(r) - \overline{v}(r)| \right\},$$

is called the distance between u and v.

It is shown that (E^1, D) is a complete metric space, see [7].

Definition 6. A function $f : R^2 \to E^1$ is called a fuzzy function in two-dimensional space. f is said to be continuous, if for arbitrary fixed $t_0 \in R^2$ and $\varepsilon > 0$ a $\delta > 0$ exists such that

$$\|t - t_0\| < \delta \Rightarrow D\left(f(t), f(t_0)\right) < \varepsilon, \qquad t = (x,y),\ t_0 = (x_0, y_0).$$

Definition 7. Let $f : [a,b] \times [c,d] \to E^1$.
For each partition $p = \{x_1, x_2, \ldots, x_m\}$ of $[a,b]$ and $q = \{y_1, y_2, \ldots, y_n\}$ of

$[c, d]$ and for arbitrary $\xi_i : x_{i-1} \leq \xi_i \leq x_i$, $2 \leq i \leq m$, and for arbitrary $\eta_j : y_{j-1} \leq \eta_j \leq y_j$, $2 \leq j \leq n$, let

$$R_p = \sum_{i=2}^{m} \sum_{j=2}^{n} f(\xi_i, \eta_j)(x_i - x_{i-1})(y_j - y_{j-1}).$$

The definite integral of $f(x, y)$ over $[a, b] \times [c, d]$ is

$$\int_c^d \int_a^b f(x, y) dx dy = \lim R_p,$$

$$(\max_{2 \leq i \leq m} |x_i - x_{i-1}|, \max_{2 \leq j \leq n} |y_j - y_{j-1}|) \to (0, 0),$$

provided that this limit exists in metric D.

If the function $f(x, y)$ is continuous in the metric D, it's definite integral exists [8]. Furthermore

$$\left(\underline{\int_c^d \int_a^b f(x, y, r) dx dy} \right) = \int_c^d \int_a^b \underline{f}(x, y, r) dx dy,$$

$$\left(\overline{\int_c^d \int_a^b f(x, y, r) dx dy} \right) = \int_c^d \int_a^b \overline{f}(x, y, r) dx dy.$$

3. Two-Dimensional Fuzzy Integral Equation

The linear two-dimensional fuzzy Fredholm integral equation of the second kind (2D-FFIE-2) is

$$u(x, y) = f(x, y) + \int_c^d \int_a^b k(x, y, s, t) u(s, t) ds dt, \quad (x, y) \in V, \tag{1}$$

where $u(x, y)$ and $f(x, y)$ are fuzzy functions on $V = [a, b] \times [c, d]$ and $k(x, y, s, t)$ is an arbitrary kernel function over $S = [a, b] \times [c, d] \times [a, b] \times [c, d]$, and u is unknown on V.

Now, we introduce parametric form of a 2D-FFIE-2 with respect to Definition 2. Let $(\underline{f}(x, y, r), \overline{f}(x, y, r))$ and $(\underline{u}(x, y, r), \overline{u}(x, y, r))$, $0 \leq r \leq 1$,

$(x, y) \in V$, be parametric form of $f(x, y)$ and $u(x, y)$, respectively. Then parametric form of 2D-FFIE-2 is as follows:

$$\begin{array}{l} \underline{u}(x,y,r) = \underline{f}(x,y,r) + \int_c^d \int_a^b \upsilon_1(x,y,s,t,\underline{u}(s,t,r),\overline{u}(s,t,r))dsdt, \\ \overline{u}(x,y,r) = \overline{f}(x,y,r) + \int_c^d \int_a^b \upsilon_2(x,y,s,t,\underline{u}(s,t,r),\overline{u}(s,t,r))dsdt, \end{array} \tag{2}$$

$$\upsilon_1(x,y,s,t,\underline{u}(s,t,r),\overline{u}(s,t,r)) = \begin{cases} k(x,y,s,t)\underline{u}(s,t,r) \; k(x,y,s,t) \geq 0, \\ k(x,y,s,t)\overline{u}(s,t,r) \; k(x,y,s,t) < 0, \end{cases}$$

and

$$\upsilon_2(x,y,s,t,\underline{u}(s,t,r),\overline{u}(s,t,r)) = \begin{cases} k(x,y,s,t)\overline{u}(s,t,r) \; k(x,y,s,t) \geq 0, \\ k(x,y,s,t)\underline{u}(s,t,r) \; k(x,y,s,t) < 0, \end{cases}$$

for each $a \leq x \leq b$ and $c \leq y \leq d$ and $0 \leq r \leq 1$. We can see that (2) is a system of linear Fredholm integral equations of the second kind with three variables in crisp case.

4. Modified Homotopy Perturbation Method

In this section, the homotopy perturbation method is used for solving (1). Suppose that the kernel function k has a degenerate form, and for convinence, we get $k(x, y, s, t) = g(x, y)h(s, t)$.

First, consider $k(x, y, s, t) \geq 0$ on S, so the parametric form of (1) is:

$$\underline{u}(x,y,r) = \underline{f}(x,y,r) + \int_c^d \int_a^b k(x,y,s,t)\underline{u}(s,t,r)dsdt, \tag{3}$$

$$\overline{u}(x,y,r) = \overline{f}(x,y,r) + \int_c^d \int_a^b k(x,y,s,t)\overline{u}(s,t,r)dsdt, \tag{4}$$

MHPH is applied for equations (3) and (4) respectively.

To explain this method for (3), we reconstitute this equation as:

$$L(\underline{u}) = \underline{u}(x,y,r) - \underline{f}(x,y,r) - \int_c^d \int_a^b k(x,y,s,t)\underline{u}(s,t,r)dsdt = 0, \tag{5}$$

we define a convex homotopy $H(\underline{u}, p, m)$ by

$$H(\underline{u},p,m) = (1-p)F(\underline{u}) + pL(\underline{u}) + mp(1-p)g(x,y) = 0, \quad p \in [0,1], \tag{6}$$

where $F(\underline{u}) = \underline{u}(x,y,r) - \underline{f}(x,y,r)$, and $F(\underline{u}) = 0$ is a trivial problem with an known solution, say $\underline{u_0}(x,y,r)$. And m is an unknown real number. Obviously,

$$H(\underline{u},0,m) = F(\underline{u}), \quad H(\underline{u},1,m) = L(\underline{u}),$$

and changing the parameter p from 0 to 1 is the same as changing $\underline{u}(x,y,r)$ from $\underline{u_0}(x,y,r)$ to a solution of $L(\underline{u}) = 0$. We can assume that the solution of (6) can be expressed as a series in p as follows:

$$\underline{u}(x,y,r) = \underline{u_0}(x,y,r) + p\underline{u_1}(x,y,r) + p^2\underline{u_2}(x,y,r) + \dots \tag{7}$$

if the series (7) is convergent as $p \to 1$, the approximate solution of (3) is

$$\underline{v}(x,y,r) = \lim_{p\to 1} \underline{u}(x,y,r) = \underline{u_0}(x,y,r) + \underline{u_1}(x,y,r) + \underline{u_2}(x,y,r) + \dots \tag{8}$$

Substituting (7) in (6), and equating the terms with the identical powers of p, we have

$$\begin{aligned}
p^0 &: \underline{u_0}(x,y,r) = \underline{f}(x,y,r),\\
p^1 &: \underline{u_1}(x,y,r) = g(x,y)\int_c^d\int_a^b h(s,t)\underline{u_0}(s,t,r)dsdt - mg(x,y)\\
&\qquad = (c-m)g(x,y), \qquad c = \int_c^d\int_a^b h(s,t)\underline{f}(s,t,r)dsdt,\\
p^2 &: \underline{u_2}(x,y,r) = g(x,y)\int_c^d\int_a^b h(s,t)\underline{u_1}(s,t,r)dsdt + mg(x,y),\\
&\qquad = (m+(c-m)\alpha)g(x,y), \qquad \alpha = \int_c^d\int_a^b k(s,t,s,t)dsdt,\\
p^{k+1} &: \underline{u_{k+1}}(x,y,r) = g(x,y)\int_c^d\int_a^b h(s,t)\underline{u_k}(s,t,r)dsdt, \qquad k \ge 2.
\end{aligned} \tag{9}$$

Now, we try to find m in such a way that $\underline{u_2}(x,y,r) = 0$. So that setting $m + (c-m)\alpha = 0$, obtain

$$m = \frac{c\alpha}{\alpha - 1}\ , \tag{10}$$

where, provided that $\alpha \neq 1$. But $\underline{u_2}(x,y,r) = 0$ implies that

$$\underline{u_3}(x,y,r) = \underline{u_4}(x,y,r) = \dots = 0,$$

so, the approximate solution of (3) is

$$\begin{aligned}
\underline{v}(x,y,r) &= \underline{u_0}(x,y,r) + \underline{u_1}(x,y,r)\\
&= \underline{f}(x,y,r) + \beta\int_c^d\int_a^b k(x,y,s,t)\underline{f}(s,t,r)dsdt,
\end{aligned} \tag{11}$$

where

$$\beta = \frac{-1}{\int_c^d \int_a^b k(s,t,s,t)dsdt - 1}\ , \tag{12}$$

and provided that $\int_c^d \int_a^b k(s,t,s,t)dsdt \neq 1$.

With the same procedure, we can obtain the approximate solution of (4) as follows:

$$\overline{\upsilon}(x,y,r) = \overline{f}(x,y,r) + \beta \int_c^d \int_a^b k(x,y,s,t)\overline{f}(s,t,r)dsdt. \tag{13}$$

So we obtain the approximate solution $\upsilon = (\underline{\upsilon}(x,y,r), \overline{\upsilon}(x,y,r))$ for (1).

In the following proposition, we bring the sufficient condition for performing υ in Lemma 4

Proposition 8. *If $\int_c^d \int_a^b k(s,t,s,t)dsdt < 1$, then the family $\upsilon(x,y)(r) = (\underline{\upsilon}(x,y,r), \overline{\upsilon}(x,y,r))\,,(x,y) \in V, r \in [0,1]$, given by (11) and (13) satisfy in Lemma 4.*

Proof. First, let $(x,y) \in V, r_1, r_2 \in [0,1], r_1 \leq r_2$, since f is a fuzzy function we have

$$\underline{f}(x,y,r_1) \leq \underline{f}(x,y,r_2) \leq \overline{f}(x,y,r_2) \leq \overline{f}(x,y,r_1), \tag{14}$$

because K is a positive function, we have $0 \leq \int_c^d \int_a^b k(s,t,s,t)dsdt < 1$, so from (12), obtain $\beta > 0$. Therefore

$$\begin{aligned}&\beta \int_c^d \int_a^b k(x,y,s,t)\underline{f}(s,t,r_1)dsdt \leq \beta \int_c^d \int_a^b k(x,y,s,t)\underline{f}(s,t,r_2)dsdt \leq \\ &\beta \int_c^d \int_a^b k(x,y,s,t)\overline{f}(s,t,r_2)dsdt \leq \beta \int_c^d \int_a^b k(x,y,s,t)\overline{f}(s,t,r_1)dsdt,\end{aligned} \tag{15}$$

by adding the inequality (14) and (15), we have

$$\underline{\upsilon}(x,y,r_1) \leq \underline{\upsilon}(x,y,r_2) \leq \overline{\upsilon}(x,y,r_2) \leq \overline{\upsilon}(x,y,r_1), \tag{16}$$

therefore the condition 1 from Lemma 4 holds.

Second, let (r_k) be a non-decreasing converging sequence converges to $r \in [0,1]$. As f is a fuzzy function and integral is continuous, condition 2 from Lemma 4 holds and the proof is completed. □

Now, assume that $k(x,y,s,t) < 0$ on S, the parametric form of (1) is:

$$\begin{aligned}\underline{u}(x,y,r) &= \underline{f}(x,y,r) + \int_c^d \int_a^b k(x,y,s,t)\overline{u}(s,t,r)dsdt,\\ \overline{u}(x,y,r) &= \overline{f}(x,y,r) + \int_c^d \int_a^b k(x,y,s,t)\underline{u}(s,t,r)dsdt,\end{aligned} \tag{17}$$

the method is applied for the system of equations (17). We choose the convex homotopy with components:

$$H_1(\underline{u}, \overline{u}, p) = (1-p)F_1(\underline{u}) + pL_1(\underline{u}, \overline{u}) + p(1-p)m_1 g(x,y) = 0,$$
$$H_2(\underline{u}, \overline{u}, p) = (1-p)F_2(\overline{u}) + pL_2(\underline{u}, \overline{u}) + p(1-p)m_2 g(x,y) = 0, \tag{18}$$

where, $p \in [0,1]$ and m_1, m_2 are unknown real numbers and

$$\begin{aligned} F_1(\underline{u}) &= \underline{u}(x,y,r) - \underline{f}(x,y,r), \\ F_2(\overline{u}) &= \overline{u}(x,y,r) - \overline{f}(x,y,r), \\ L_1(\underline{u}, \overline{u}) &= \underline{u}(x,y,r) - \underline{f}(x,y,r) - \int_c^d \int_a^b k(x,y,s,t)\overline{u}(s,t,r)dsdt, \\ L_2(\underline{u}, \overline{u}) &= \overline{u}(x,y,r) - \overline{f}(x,y,r) - \int_c^d \int_a^b k(x,y,s,t)\underline{u}(s,t,r)dsdt. \end{aligned} \tag{19}$$

We define solution of (18) as

$$\underline{u}(x,y,r) = \underline{u_0}(x,y,r) + p\underline{u_1}(x,y,r) + p^2\underline{u_2}(x,y,r) + \dots$$
$$\overline{u}(x,y,r) = \overline{u_0}(x,y,r) + p\overline{u_1}(x,y,r) + p^2\overline{u_2}(x,y,r) + \dots \tag{20}$$

So, the approximate solution of (17) is

$$\underline{v}(x,y,r) = \lim_{p\to 1} \underline{u}(x,y,r) = \underline{u_0}(x,y,r) + \underline{u_1}(x,y,r) + \dots$$
$$\overline{v}(x,y,r) = \lim_{p\to 1} \overline{u}(x,y,r) = \overline{u_0}(x,y,r) + \overline{u_1}(x,y,r) + \dots \tag{21}$$

By substituting (20) into (18), and equating therms with the identical powers of p, we have

$$\begin{aligned} p^0 &: \underline{u_0}(x,y,r) = \underline{f}(x,y,r), \\ &\quad \overline{u_0}(x,y,r) = \overline{f}(x,y,r), \\ p^1 &: \underline{u_1}(x,y,r) = (c_1 - m_1)g(x,y), \qquad c_1 = \int_c^d \int_a^b h(s,t)\overline{f}(s,t,r)dsdt, \\ &\quad \overline{u_1}(x,y,r) = (c_2 - m_2)g(x,y), \qquad c_2 = \int_c^d \int_a^b h(s,t)\underline{f}(s,t,r)dsdt, \\ p^2 &: \underline{u_2}(x,y,r) = (m_1 + (c_2 - m_2)\alpha)g(x,y), \\ &\quad \overline{u_2}(x,y,r) = (m_2 + (c_1 - m_1)\alpha)g(x,y), \qquad \alpha = \int_c^d \int_a^b k(s,t,s,t)dsdt, \\ p^{k+1} &: \underline{u_{k+1}}(x,y,r) = g(x,y)\int_c^d \int_a^b h(s,t)\overline{u_k}(s,t,r)dsdt, \\ &\quad \overline{u_{k+1}}(x,y,r) = g(x,y)\int_c^d \int_a^b h(s,t)\underline{u_k}(s,t,r)dsdt, \qquad k \geq 2. \end{aligned} \tag{22}$$

Now, we find parameters m_1, m_2 such that $\underline{u_2}(x,y,r) = 0$ and $\overline{u_2}(x,y,r) = 0$.

So, we have the following linear system of equations

$$\begin{aligned} m_1 + (c_2 - m_2)\alpha = 0, \\ m_2 + (c_1 - m_1)\alpha = 0, \end{aligned} \tag{23}$$

by solving the above system, we obtain

$$\begin{aligned} m_1 = -(c_2\alpha + c_1\alpha^2)/(1-\alpha^2), \\ m_2 = -(c_1\alpha + c_2\alpha^2)/(1-\alpha^2). \end{aligned} \tag{24}$$

Therefore, the approximate solution of (17) is

$$\begin{aligned} \underline{v}(x,y,r) &= \underline{u_0}(x,y,r) + \underline{u_1}(x,y,r) \\ &= \underline{f}(x,y,r) + \tfrac{1}{1-\alpha^2}\int_c^d \int_a^b k(x,y,s,t)(\overline{f}(s,t,r) + \alpha \underline{f}(s,t,r))dsdt, \\ \overline{v}(x,y,r) &= \overline{u_0}(x,y,r) + \overline{u_1}(x,y,r) \\ &= \overline{f}(x,y,r) + \tfrac{1}{1-\alpha^2}\int_c^d \int_a^b k(x,y,s,t)(\underline{f}(s,t,r) + \alpha \overline{f}(s,t,r))dsdt, \end{aligned} \tag{25}$$

where provided that $(\int_c^d \int_a^b k(s,t,s,t)dsdt)^2 \neq 1$.

Then $v = (\underline{v}(x,y,r), \overline{v}(x,y,r))$ is the approximate solution for (1).

5. Numerical Results

Example 9. Consider the following two-dimensional fuzzy Fredholm integral equation [1]:

$$u(x,y) = f(x,y) + \int_0^1 \int_0^1 x^2 ysu(s,t)dsdt, \quad 0 \le x,y \le 1,$$

where

$$f(x,y)(r) = \left(x\sin(y/2)(r^2+r), x\sin(y/2)(4-r^3-r)\right), \quad 0 \le r \le 1.$$

By the direct method, we have the exact solution

$$\begin{aligned} \underline{u}(x,y,r) &= \left(x\sin(y/2) - 16/21\,(\cos(1/2)-1)\,x^2y\right)(r^2+r), \\ \overline{u}(x,y,r) &= \left(x\sin(y/2) - 16/21\,(\cos(1/2)-1)\,x^2y\right)(4-r^3-r), \end{aligned}$$

and by using the homotopy perturbation method, we yield the exact solution. Figure 1 shows the solutions with homotopy perturbation method and direct method at points $(0.5, 0.4, r)$, where $r \in [0, 1]$.

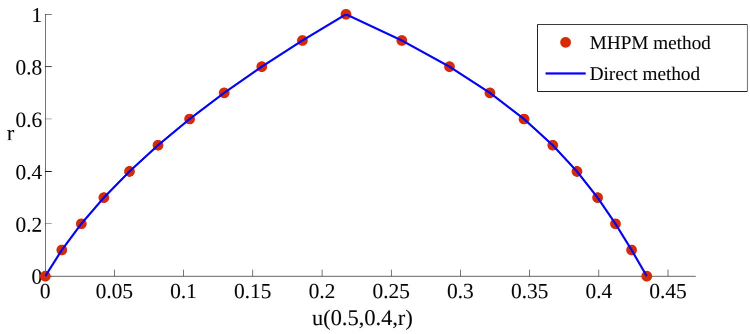

Figure 1: The results of Example 9

Example 10. For the following 2D-FFIE-2:

$$u(x, y) = f(x, y) + \int_0^1 \int_0^1 \pi xy \sin(2\pi s) u(s, t) ds dt, \quad 0 \le x, y \le 1,$$

where

$$\underline{f}(x, y, r) = \pi xy\left(\frac{13}{15}(r^2 + r) + \frac{2}{15}(4 - r^3 - r)\right),$$

$$\overline{f}(x, y, r) = \pi xy\left(\frac{2}{15}(r^2 + r) + \frac{13}{15}(4 - r^3 - r)\right), \quad 0 \le r \le 1.$$

The modified homotopy perturbation method gives exact solution

$$\underline{u}(x, y, r) = \pi xy\left(\frac{5}{15}r^3 + \frac{17}{15}r^2 + \frac{22}{15}r - \frac{20}{15}\right),$$

$$\overline{u}(x, y, r) = -\pi xy\left(\frac{17}{15}r^3 + \frac{5}{15}r^2 + \frac{22}{15}r - \frac{68}{15}\right).$$

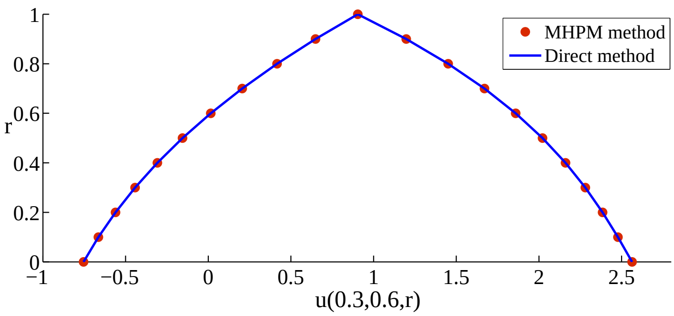

Figure 2: The results of Example 10

6. Conclusion

In this paper the modified homotopy perturbation method is applied for solving two-dimensional fuzzy Fredholm integral equation of the second kind. To check the method, it is applied to different problems with known exact solutions. The numerical results confirm the validity and the low cost of the method, and suggest it is a viable alternative to existing numercal method for solving the problem under consideration.

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