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"ON A CLASS OF IRREDUCIBLE REPRESENTATIONS OF THE BRAID GROUP, B_N"

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Abstract: A calculation mistake occured in the computations of Wada's representation in Section 2 [1, p.683]; which results in wrong theorems in Section 3 of [1, pp.684-688], namely Theorems 4,5,6 and 7. Having fixed these mistakes, we replace Definition 2 of Section 2 and the wrong theorems of Section 3 of that previous work by the following definition and theorems.

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1. Definition

Definition 2. The representation, discovered by M. Wada, asserts that the

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automorphism corresponding to σ_i takes x_i to $x_i^2 x_{i+1}$, x_{i+1} to $x_{i+1}^{-1} x_i^{-1} x_{i+1}$ and fixes all other generators.

Let F_n be the free group of rank n with free basis $x_1, \ldots, x_n$. It is easy to see that $F_n =< g_1, \ldots, g_n >$, where $g_1 = x_1$, $g_2 = g_1 x_2$, $\ldots$, $g_n = g_{n-1} x_n$. The action of the braid generator σ_i on the basis $\{g_1, \ldots, g_n\}$ is given by

$$\sigma_1 : \begin{cases} g_1 \to g_1 g_2, & \\ g_j \to g_j, & \text{if } j \neq 1. \end{cases}$$

For $1 < i < n$, we have that

$$\sigma_i : \begin{cases} g_i \to g_i g_{i-1}^{-1} g_{i+1}, & \\ g_j \to g_j, & \text{if } j \neq i. \end{cases}$$

Let $\rho : \mathbb{Z}[F_n] \to \mathbb{Z}[t^{\pm 1}]$, where $\mathbb{Z}[t^{\pm 1}]$ is the ring of Laurent polynomials with independent indeterminate t. The map ρ is defined as

$$\rho(g_i) = \begin{cases} 1, & \text{if } i \text{ is even}, \\ t, & \text{if } i \text{ is odd}. \end{cases}$$

Using Magnus representation of subgroups of the automorphism group of the free group $F_n = \{g_1, \ldots, g_n\}$, we determine Wada's representation α: $B_n \to GL_n(\mathbb{Z}[t^{\pm 1}])$. The images of the generators under Wada's representation are given by

$$\alpha(\sigma_1) = \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix} \oplus I_{n-2},$$

$$\alpha(\sigma_k) = I_{k-2} \oplus \begin{pmatrix} 1 & 0 & 0 \\ -t & 1 & t \\ 0 & 0 & 1 \end{pmatrix} \oplus I_{n-k-1}, \text{ if } k \text{ is odd } (1 < k < n),$$

and

$$\alpha(\sigma_k) = I_{k-2} \oplus \begin{pmatrix} 1 & 0 & 0 \\ -t^{-1} & 1 & t^{-1} \\ 0 & 0 & 1 \end{pmatrix} \oplus I_{n-k-1}, \text{ if } k \text{ is even } (1 < k < n).$$

2. Reducibility of Wada's Representation

Theorem 4. *Wada's representation, namely α: $B_n \rightarrow GL_n(\mathbb{C})$, is a reducible representation.*

Proof. Let u be the column vector in $\mathbb{C}^n$ defined as

$$u = \begin{cases} (1,0,1,0,\dots,1,0)^T & \text{if } n \text{ is even,} \\ (1,0,1,0,\dots,0,1)^T & \text{if } n \text{ is odd.} \end{cases}$$

It is easy to see that the subspace generated by u is invariant under the representation α because $\alpha(\sigma_k)(u) = u$ for every $k \in \{1,2,\dots,n-1\}$. □

References

[1] M.N. Abdulrahim, Madline Al Tahan, On a class of irreducible representations of the Braid Group, B_n, *International Journal of Applied Mathematics*, **23**, No. 4 (2010), 681-691.