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A NOTE OF OPTIMAL LINEAR CONTROL TECHNICAL APPLIED IN THE CHAOS STABILIZATION IN THE MATHIEU-VAN DER POL AUTONOMOUS OSCILLATOR

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Abstract: This paper presents the application of the Linear Optimal Control Technical to control the chaotic movement of a hyperchaotic system with three positive Lyapunov exponents. The Mathieu - van der Pol autonomous system is demonstrated with a chaotic behavior and we present the control technical for the chaos stabilization. The simulation results show the effectiveness of the control strategy.

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1. Introduction

A new four order system – Mathieu-van der Pol autonomous system is proposed by [1]. In this work, the authors present a new hyperchaotic system with three positive Lyapunov exponents (called Tri-Chaos), as well as the implementation of an electronic circuit.

Chaos control problems consist of attempts to stabilize a chaotic system to an equilibrium point, a periodic orbit, or more general, about a given reference trajectory. In the last years, a significant interest in control of the nonlinear systems, exhibiting chaotic behavior, has been observed and many of the techniques were discussed in the literature recently, see [2]-[11]. The aim of this paper is to propose the application of the Linear Optimal Control [11] to control the chaotic movement of a hyperchaotic system with three positive Lyapunov exponents proposed by Shih-Yu et al. [1].

The paper is organized as follows: in Section 2 we demonstrate the mathematical model and show the nonlinear dynamics of the model. In Section 3, we discuss a control design problem for non-linear model. In Section 4, we give the concluding remarks. Finally, we list out the bibliographic references used in this paper.

2. Non-Linear Model

The work [1] introduced a new hiperchaotic system with three positive Lyapunov expoents, also tri-chaos call. Via linear coupling, Mathieu, and van der Pol systems are coupled with each other and then become a new four order system-Mathieu-van der Pol autonomous system. The authors propose a new chaotic system with three Lyapunov exponents, and the governing equations of system, Mathieu-van der Pol autonomous system, are:

$$\begin{aligned} \dot{x_1} &= x_2, \\ \dot{x_2} &= -(a+bx_3)x_1 - (a+bx_3){x_1}^3 - cx_2 + dx_3, \\ \dot{x_3} &= x_4, \\ \dot{x_4} &= -ex_3 + f(1-{x_3}^2)x_4 + gx_1, \end{aligned} \tag{1}$$

where x_1, x_2, x_3 and x_4 are four states of the system, a, b, c, d, e, f and g are the parameters of the Mathieu-van der Pol system, [1].

We choose the chaotic behavior for analyzing when the parameters $a = 91.7$, $b = 5.023$, $c = 0.01$, $d = 91$, $e = 87.001$, $f = 18$ and $g = 9.5072$, see [1].

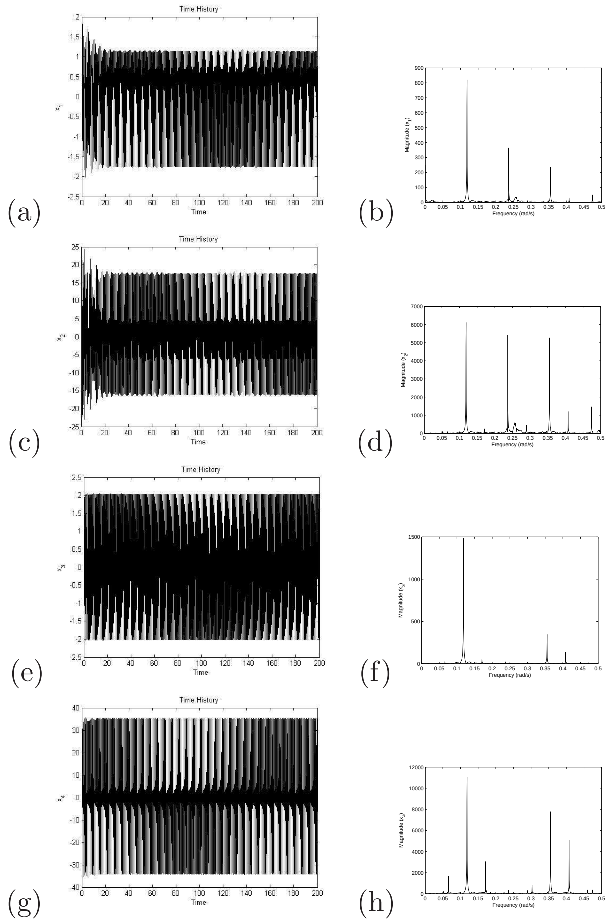

Figure 1: (a)Dynamical behavior of the time history (x_1), (b) FFT for (x_1), (c) Dynamical behavior of the time history (x_2), (d) FFT for (x_2), (e) Dynamical behavior of the time history (x_3), (f) FFT for (x_3), (g) Dynamical behavior of the time history (x_4), and (h) FFT for (x_4).

Figures 1 show the dynamical behavior of the time history and FFTs for states of the system, respectively.

Figures 2 show the portrait phase for the chaotic behavior.

Figure 3 illustrates the four Lyapunov exponents, $\lambda_1 = 12.126062$; $\lambda_2 = 7.7712$; $\lambda_3 = 7.7814$ and $\lambda_4 = -9.6868$, demonstrating the presence of the Tri-Chaos with the three Lyapunov exponent positive. The total time for Lyapunov

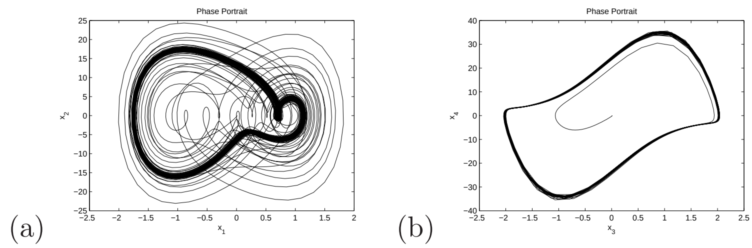

Figure 2: Phase Portrait. (a) x_1 and x_2 projections and (b) x_3 and x_4 projections

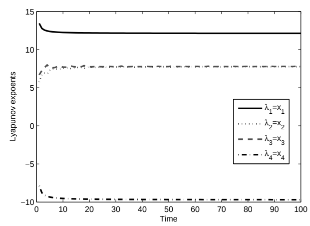

Figure 3: Dynamics of Lyapunov exponents for system

exponents computation was $\Delta\tau = 100,000$ with time-step of $\tau = 0.5$.

3. Control Design

In this section we develop an optimal linear control design for the systems considered, i.e. the Mathieu-van der Pol autonomous system, reducing the chaotic movement to a stable orbit.

Next, we present a summary of the theory of the used methodology. Due to the simplicity in configuration and implementation, the linear state feedback control is especially attractive, see e.g. [11]. It is used before on some mathematical models, as in [6]-[10], undeserved others. We remarked that this approach is analytical, without dropping any non-linear term.

Let the nonlinear governing equations of motion (1) be rewritten in a state form:

$$\dot{x} = Ax + g(x) + U. \tag{2}$$

If one considers a vector function $\tilde{x}$, that characterizes the desired trajectory, and takes the control U vector consisting of two parts: $\tilde{u}$ being the feed forward and u_f is a linear feedback, in such a way it gives

$$u_f = Bu, \tag{3}$$

where B is a constant matrix. Next, by taking the deviation of the trajectory of system (2) to the desired one $y = x - \tilde{x}$.

It is optimal, in order to transfer the non-linear system (3) from any initial to final state $y(t_f) = 0$, minimizing the functional $\tilde{J} = \frac{1}{2}\int_0^\infty (y^T \tilde{Q} y + u^t R u)dt$, where the symmetric matrix $P(t)$ is evaluated through the solution of the matrix Ricatti differential equation

$$PA + A^T P - PBR^{-1}B^T P + Q = 0, \tag{4}$$

satisfying the final condition $P(t_f) = 0$.

In addition, with the feedback control (3), there exists a neighborhood $\Gamma_0 \subset \Gamma, \Gamma \subset \Re^n$, of the origin such that if $x_0 \in \Gamma_0$, the solution $x(t) = 0$, $t \geq 0$ of the controlled system (4) is locally asymptotically stable, and $J_{min} = x_0^T P(0) x_0$. Finally, if $\Gamma = \Re^n$, then the solution $y(t) = 0, t > 0$ of the controlled system (4) is globally asymptotically stable.

Using the theorem from [12], the dynamic error y can be minimized ($y \to 0$).

3.1. Theorem Linear Optimal Control

If there are matrices Q and R, positive definite, Q symmetric, and such that the matrix

$$\tilde{Q} = Q - G^T(x, \tilde{x})P - PG(x, \tilde{x}) \tag{5}$$

is positive definite for the limited matrix G, then the linear feedback control

$$u = -R^{-1}B^T P_y \tag{6}$$

is optimal, in order to drive the non-linear system (3) of any initial state to the terminal state

$$y(\infty) = 0, \tag{7}$$

minimizing the functional

$$J = \int_0^\infty (y^T \tilde{Q} y + u^T R u)dt, \tag{8}$$

where the symmetric matrix P is calculated from the nonlinear Riccati equation:

$$PA + A^T P - PBR^{-1}B^T P + Q = 0. \tag{9}$$

Next, we will apply this methodology to model (1).

3.2. Application of the Linear Optimal Control to the System

The equations (1) describe the system:

$$\begin{aligned} \dot{x_1} &= x_2 + U, \\ \dot{x_2} &= -(a+bx_3)x_1 - (a+bx_3){x_1}^3 - cx_2 + dx_3, \\ \dot{x_3} &= x_4, \\ \dot{x_4} &= -ex_3 + f(1-{x_3}^2)x_4 + gx_1, \end{aligned} \tag{10}$$

where the function of control U is defined in the equation (2).

We will obtain $B = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$, $y = \begin{pmatrix} x_1 - \tilde{x}_1 \\ x_2 - \tilde{x}_2 \\ x_3 - \tilde{x}_3 \\ x_4 - \tilde{x}_4 \end{pmatrix}$, $\tilde{x} = \begin{pmatrix} 0.01sin(\pi t) \\ 0.01sin(\pi t) \\ 0.01sin(\pi t) \\ 0.01sin(\pi t) \end{pmatrix}$, $Q = I_4$, $A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -92.7777 & -0.01 & 90.9497 & 0 \\ 0 & 0 & 0 & 0 \\ 9.5071 & 0 & -87.0046 & 17.9982 \end{pmatrix}$, where the controllability matrix R of the system to the pair $[A, B]$ is obtained by $R = [B|AB|A^2B|A^3B] \neq 0$.

Thus, $R = \begin{pmatrix} 1 \end{pmatrix}$.

Then the matrix $P(t)$ is done by

$$P = \begin{pmatrix} 0.4349 & -0.0018 & -0.0448 & -0.0135 \\ -0.0018 & 0.0043 & 0.0110 & -0.0056 \\ -0.4482 & 0.0110 & 1.1717 & -0.1709 \\ -0.0135 & -0.0056 & -0.1709 & 0.0578 \end{pmatrix}$$

and (an optimal control)

$$u = -2.8743x_1 + 0.7939x_2 + 56.3594x_3 - 13.2351x_4.$$

The trajectories of the system, without and with control may be seen through Figures 4. According to the optimal control verification [12], the function (4) is numerically calculated across $L(t) = y^T \tilde{Q} y$, where $L(t)$ is defined positive and it is show in Figure 5.

4. Conclusions

In this work the presence of chaotic behavior is verified in the proposed system. It is mathematically modeled by means of a set of four ordinary differential equations.

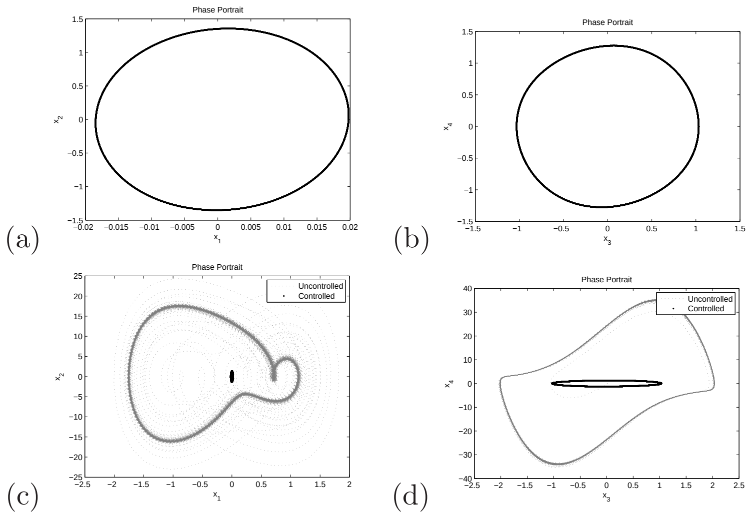

Figure 4: Phase Portrait. (a) x_1 and x_2 controlled projections, (b) x_3 and x_4 controlled projections, (c) x_1 and x_2 uncontrolled and controlled projections, (d) x_3 and x_4 uncontrolled and controlled projections.

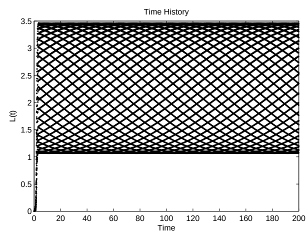

Figure 5: Controlled dynamical behavior of the time history (time 0-200).

We also propose to use an optimal linear feedback control strategy, applied to mathematical model proposed. This kind of control strategy is reducing the chaotic movement of the system to a small stable orbit.

Figures 4 and 5 illustrate the effectiveness of the control strategy to the Mathieu-van der Pol autonomous system.

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