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MONTE CARLO METHOD FOR PRICING SOME PATH DEPENDENT OPTIONS

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Abstract: The numerical methods form an important part of pricing options, especially in cases where there is no closed form analytic formula.

The Monte Carlo method is one of the primary numerical methods that is currently used by financial professionals for determining the price of options and security pricing problems with emphasis on improvement in efficiency.

We discuss the pricing of exotic options with special emphasis on path dependent options, like Asian and lookback options. Monte Carlo simulation technique is very versatile in cases where there is no closed form analytical formula. This method is slow and time consuming but very flexible even for multi-dimensional problems and has proved to be a valuable and flexible computational tool in modern finance.

We compare the result of the Monte Carlo method with the analytic Black-Scholes results and the exact values of the options.

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1. Introduction

In the last few decades, options have undergone a transformation from specialized and obscure securities to ubiquitous components of the portfolio of not only large fund managers, but ordinary investors.

An option is a financial contract or a contingent claim that gives the holder the right, but not the obligation to buy or sell an underlying asset for a predetermined price called the strike or exercise price during a certain period of time. Options fall into two classes: a call which is the right to buy the underlying asset and a put which entitles the owner to sell the underlying asset. An American option allows exercise at any point during the life of the option. A European option allows exercise to occur only at expiration. A path dependent option is an option whose value depends on the sequence of prices of the underlying asset during the whole or part of the option's life rather than just the final price of the asset. Examples of path dependent option are Asian options in which the underlying variable is the average price over a period of time. Because of this, Asian options have a lower volatility and hence rendering them cheaper relative to their European counterparts. The Lookback option is defined as an option whose strike price corresponds to the minimum and maximum price recorded by the underlying asset during the option's life. Obviously, the more flexible the option, the more expensive it will be.

Black and Scholes published their seminal work on option pricing [2] in which they described a mathematical frame work for finding the fair price of a European option. They used a no-arbitrage argument to describe a partial differential equation which governs the evolution of the option price with respect to the maturity time and the price of the underlying asset.

In the recent years the complexity of numerical computational in financial theory and practice has increased greatly, putting more demands on computation speed and efficiency.

Numerical methods are needed for pricing options in cases where analytic solutions are either unavailable or not easily computable. They are used for a variety of purpose in finance. These include the valuation of securities, the estimation of their sensitivities, risk analysis and stress testing of portfolios.

The subject of numerical methods in the area of options valuation and hedging is very broad. A wide range of different types of contracts are available and in many cases there are several candidate models for the stochastic evolution of the underlying state variables [14].

Now, we present an overview of Monte Carlo approach in the context of Black-Scholes-Merton [2, 9] introduced by Boyle [3] for pricing some path dependent options. Brennan and Schwarz [5] introduced finite difference methods for pricing derivative through the solution of some partial differential equations. The sufficient conditions for dynamic stability and convergence to equilibrium of the growth rate of the value function of stock shares were given by Osu [8] and binomial model for pricing options based on risk-neutral valuation was derived by Cox-Ross-Rubinstein [6]. Monte Carlo method is a useful tool and provides much of the infrastructure in which many contributions to the field over the past two decades have been centered.

In this work we consider the pricing of some path dependent options namely Asian option and Lookback option by the use of Monte Carlo method.

2. Numerical Methods for Options Valuation

This section presents one popular numerical method for path dependent options valuation namely:

- Monte Carlo Method.

2.1. Monte Carlo Method, see [9]

The Monte Carlo method is a numerical method that is useful in many situations when no closed form solution is available. It is simple and easy to implement.

The "Monte Carlo" was introduced by Von Neumann and Ulam during world war II (1940), as a code for the secret work at Los Alamos. The standard Monte Carlo technique uses random numbers which are independent random variables uniformly distributed over the unit interval [0, 1).

The Monte Carlo simulation has been applied in many fields, including the pricing of financial derivatives. This method can be used in estimating option prices for derivatives that do or do not have a convenient analytical formula. The risk neutral world is calculated using a sampling procedure and discounted at the risk-free interest rate. In an efficient market the pricing of an option is equivalent to evaluating the expectation of its discounted payoff under a specified measure.

Boyle [3] was the first researcher to introduce Monte Carlo simulation into finance. Twenty years later, Boyle, Broadie and Glasserman described in [4] research advances that improved efficiency and broadened the types of problem where simulation can be applied. The research undertaken has proved that simulation is a valuable tool for pricing options.

In our analysis, we make the usual assumptions underlying the Black-Scholes-Merton, in that

- The price of the underlying asset of an option follows a log-normal random walk.
- There are no arbitrage opportunities.
- The price of the underlying asset is expected to appreciate at the risk-free rate of interest.

A Monte Carlo simulation can be used as a procedure for sampling random outcomes of a process followed by the stock price [9]

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t), \tag{1}$$

where dW_t is a Wiener process and $S(t)$ is the stock price. If $\delta S(t)$ is the increase in the stock price in the next small interval of time δt, then

$$\frac{\delta S(t)}{S}(t) = \mu\delta t + \sigma Z\sqrt{\sigma t}, \tag{2}$$

where $Z \sim N(0,1)$, σ is the volatility of the stock price and μ is the expected return in a risk neutral world,(2) is expressed as

$$S(t+\delta t) - S(t) = \mu S(t)\delta t + \sigma S(t)Z\sqrt{\delta t}. \tag{3}$$

We can calculate the value of S at time $t+\delta t$ from the initial value of S, then the value of S at time $t+2\delta t$, from the value at $t+\delta t$ and so on. We use N random samples from a normal distribution to simulate a trial for a complete path followed by S. It is more accurate to simulate $\ln S$ than S, we transform the asset price process using Ito's lemma

$$d(\ln S) = \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma dW(t),$$

so that

$$\ln S(t+\delta t) - \ln S(t) = \left(\mu - \frac{\sigma^2}{2}\right)\delta t + \sigma Z\sqrt{\delta t},$$

or

$$S(t+\delta t) = S(t)\exp\left[\left(\mu - \frac{\sigma^2}{2}\right)\delta t + \sigma Z\sqrt{\delta t}\right]. \tag{4}$$

For example, we consider an Asian options whose Stock price process at maturity time T is given by

$$S_T^j = S \exp\left[\left(\mu - \frac{\sigma^2}{2}\right) T + \sigma Z \sqrt{T}\right], \tag{5}$$

where $j = 1, 2, ..., M$ and M denotes the number of trials or the different states of the world. These M simulations are the possible paths that a stock price can have at maturity date T. The estimated Asian call option value is

$$C = \frac{1}{M} \sum_{j=1}^{M} e^{-rT} \max[S_T^j - S_t, 0], \tag{6}$$

where S_t is the strike price determined by either arithmetic or geometric mean [4]. This is an unbiased estimate of the derivative's price. When the number of trials M is large, the central limit theorem provides a confidence interval for the estimate, biased on the sample variance of the discounted payoff. The M independent trials carried out depends on the accuracy required. If ω is the standard deviation and $\overline{\mu}$ is the mean of the discounted payoffs given by (6) then the standard error is estimated by $\frac{\omega}{\sqrt{M}}$. A 0.95 confidence interval for the price f of the derivative is therefore given by

$$\overline{\mu} - \frac{1.96\omega}{\sqrt{M}} < f < \overline{\mu} + \frac{1.96\omega}{\sqrt{M}} \tag{7}$$

under the assumption that f is normally distributed.

The Monte Carlo simulation is particularly relevant when the payoff of financial derivatives depends on the path followed by the underlying asset during the life of the, that is, for path dependent options. The method can also be applied when the value of the financial derivative depends only on the final value of the underlying asset. An example is the European style option whose payoff depends on the value of S at maturity time T [15]. The stock price process for an Asian option can also be used for pricing vanilla options.

2.2. Variance Reduction Procedures

This is a procedure used to increase the precision of the estimates that can be obtained for a given number of the iterations. Every output random variable from the simulation is associated with a variance which limits the precision of the simulation results [17].

The uncertainty about the value of the derivative is inversely proportional to the square root of the number of trials. Then, if the simulation is to give accurate results, very large number of simulated sample paths is usually necessary. This is very expensive in terms of computational time. The variance reduction technique refines and improves the efficiency of the simulation.

2.2.1. Antithetic Variance Technique

This is a variance reduction technique used in Monte Carlo methods. In this technique, a simulation trial involves calculating two values of the derivative. The first value f_1 is calculated in the usual way. The second value f_2 is calculated by changing the sign of all the random samples from the standard normal distribution [9].

For exampl,e if we use (5), then we have two equations of the form

$$S_T = S\exp\left[\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma Z\sqrt{T}\right], \tag{8}$$

$$S_T = S\exp\left[\left(\mu - \frac{\sigma^2}{2}\right)T - \sigma Z\sqrt{T}\right], \tag{9}$$

where Z is a sample used to calculate f_1 and $-Z$ is the corresponding sample used to calculate f_2. We prefer the use of random inputs obtained from the collection of antithetic pairs $(Z, -Z)$ as they are more generally distributed than a collection of $2N$ independent samples. The pair is called antithetic because they exhibit negative independence. The sample mean of the antithetic pairs always equals the population mean of zero. The mean over finitely many independent samples is almost surely different from zero. We denote f^* as the average of f_1 and f_2,

$$f^* = \frac{f_1 + f_2}{2}.$$

Then

$$\begin{aligned} Var(f^*) = Var\left(\frac{f_1+f_2}{2}\right) &= E\left[\frac{(f_1+f_2) - \mu(f_1+f_2)}{2}\right]^2 \\ &= E\left[\frac{(f_1-\mu f_1) + (f_2 - \mu f_2)}{2}\right]^2 \\ &= E\left[\frac{f_1-\mu f_1}{2}\right]^2 + 2E\left[\left(\frac{f_1-\mu f_1}{2}\right)\left(\frac{f_2-\mu f_2}{2}\right)\right] + E\left[\frac{f_2-\mu f_2}{2}\right]^2 \\ &= \frac{1}{4}Var[f_1] + \frac{1}{4}Var[f_2] + \frac{1}{2}Cov[f_1, f_2]. \end{aligned} \tag{10}$$

If the covariance $Cov[f_1, f_2]$ between f_1 and f_2 is negative, this will yield a smaller estimate of the variance than an independent estimate [8].

2.2.2. Control Variance Technique

This is the another variance reduction technique used in the Monte Carlo methods. It exploits information about the errors in estimates of known quantities to reduce the error of an estimate of an unknown quantity.

In this technique, we replace the evaluation of an unknown expectation with the evaluation of the difference between the unknown quantity and a related quantity, whose expectation is known.

The control variate uses a second estimate with a high positive correlation with the estimate of interest. We carry out two simulations using the same number streams and the same δt. Let f_X and f_Y be the respective values of X and Y. Then we can write

$$f_X = E[f_X^*]$$

and

$$f_Y = E[f_Y^*],$$

where f_X^* and f_Y^* are the estimates values of X and Y respectively.

Derivative X whose value is f_X is the security under consideration, also derivative Y whose value is f_Y is similar to derivative X and has an analytical solution available. A random variate f_Y is a control variate for f_X if it is correlated with f_X. Then

$$f_X^* = f_X^* + (f_Y - f_Y^*), \tag{11}$$

where f_Y is the known value of Y. The known error $(f_Y - f_Y^*)$ is used as a control in the estimation of f_X. The value f_X^* adjusts the estimator f_X according to the difference between the known value f_Y and the observed value f_Y^*. We aim to reduce the covariance and comparing the value of derivatives X and Y, we have

$$Var[f_X^*] = Var[f_X^*] + Var[f_Y] + Var[f_Y^*] - 2Cov[f_X^*, f_Y^*] \tag{12}$$

and

$$Var[f_Y] = 0,$$

since f_Y is the known value of Y and thus not a random variable. The control variate technique is effective if the covariance between f_X^* and f_Y^* is large, that is, if

$$2Cov[f_X^*, f_Y^*] > Var[f_X^*] + Var[f_Y^*],$$

then the variance is reduced [7].

3. Numerical Implementation

This section presents how the Monte Carlo method for pricing options can be implemented.

3.1. Procedures for the Implementation of Monte Carlo Method

The basis of Monte Carlo simulation method is the strong law of large numbers, which states that the arithmetic mean of independent, identically distributed random variables, converges towards their mean almost surely. The Monte Carlo simulation method uses the risk valuation result and lends itself naturally to the evaluation of security prices represented as expectation. The expected payoff in a risk neutral world is calculated using a sampling procedure. Generally, the main procedures are followed when using the Monte Carlo simulation.

- Simulate a path of the underlying asset (1) under the risk neutral condition within the desired time horizon
- Discount the payoff corresponding to the path at the risk-free interest rate.
- Repeat the procedure for a high number of simulated sample path
- Average the discounted cash flows over sample paths to obtain the option's value [7,9].

In effect, this method computes a multi-dimensional integral and the expected value of the discounted payoffs over the space of sample paths. The increase in the complexity of derivative securities in recent years has led to a need to evaluate high-dimensional integrals.

Now we consider an Asian option which is an example of an exotic option that has path dependent payoff and this makes it ideally suited for pricing using the Monte Carlo approach.

3.2. Asian Options

These options are commonly traded on currencies and commodity products which have low trading volumes. They were originally used in 1987 when

Banker's Trust Tokyo office used them for pricing average options on crude oil contracts and hence the name "Asian" option.

Computing an Asian option price means computing the discounted expectation of the payoff. This suggests the following algorithm to determine the Asian option price through Monte Carlo method.

We simulate M independent realization X^j of the final payoffs X given by

$$X^j_{call} = \max[S^j_T - \bar{S}_t, 0]$$

and

$$X^j_{put} = \max[\bar{S}_t - S^j_T, 0],$$

where X^j_{call} and X^j_{put} are called the payoff for the Asian call and put options respectively. The discretely monitored Asian call (put) option has the estimated value in the jth path given by

$$C^j_{call} = e^{-rT} \max[S^j_T - \bar{S}_t, 0] \tag{13}$$

and

$$C^j_{put} = e^{-rT} \max[\bar{S}_t - S^j_T, 0] \tag{14}$$

respectively, where S^j_T and $\bar{S}_t$ are called stock price at maturity time T and strike price respectively. Then the average of the underlying asset price can be calculated using equations (15) and (16) below called arithmetic and geometric average respectively,

$$S_A(t) = \frac{S(t_1) + S(t_2) + ... + S(t_N)}{N} = \frac{1}{N}\sum_{j=1}^{N} S(t_j) \tag{15}$$

and

$$S_G(t) = [S(t_1)S(t_2)...S(t_N)]^{\frac{1}{N}} = \left(\prod_{j=1}^{N} S(t_j)\right)^{\frac{1}{N}}. \tag{16}$$

The payoff of arithmetic Asian options is given by

$$\text{Payoff}_{Asian-call} = \max(S_A(t) - K, 0) = \max\left[\frac{1}{N}\sum_{j=1}^{N} S(t_j) - K, 0\right], \tag{17}$$

$$\text{Payoff}_{Asian-put} = \max(K - S_A(t), 0) = \max\left[K - \frac{1}{N}\sum_{j=1}^{N} S(t_j), 0\right]. \tag{18}$$

The payoff of geometric Asian options is given by

$$\begin{aligned}\text{Payoff}_{Asian-call} &= \max(S_A(t) - K, 0)\\ &= \max\left[\left(\prod_{j=1}^{N} S(t_j)\right)^{\frac{1}{N}} - K, 0\right],\end{aligned} \tag{19}$$

$$\begin{aligned}\text{Payoff}_{Asian-put} &= \max(K - S_G(t), 0)\\ &= \max\left[K - \left(\prod_{j=1}^{N} S(t_j)\right)^{\frac{1}{N}}, 0\right].\end{aligned} \tag{20}$$

We repeat the procedure for $j = 1, 2, ..., M$, where M denotes the number of trials. These M simulations are the possible paths that a stock price can have at maturity time T. The final estimated Asian call option value is

$$C_{call} = \frac{1}{M}\sum_{j=1}^{M} C_{call}^{j} = \frac{1}{M}\sum_{j=1}^{M} e^{-rT}\max[S_T^j - \bar{S}_t, 0] \tag{21}$$

and the corresponding Asian put option value is given by

$$C_{put} = \frac{1}{M}\sum_{j=1}^{M} C_{put}^{j} = \frac{1}{M}\sum_{j=1}^{M} e^{-rT}\max[\bar{S}_t - S_T^j, 0]. \tag{22}$$

In this work we consider a closed form pricing solution to geometric averaging options by altering the volatility. Geometric averaging options can be priced via a closed form analytic solution because of the reason that the geometric average of the underlying prices follows a lognormal distribution as well, whereas with arithmetic average rate options, this condition collapses.

The solutions to the geometric averaging Asian call and put are given respectively by:

$$C_G = Se^{(\omega - r)(T-t)}N(d_1) - Ke^{-r(T-t)}N(d_2) \tag{23}$$

and

$$P_G = Ke^{-r(T-t)}N(d_2 - Se^{(\omega - r)(T-t)}N(d_1). \tag{24}$$

Where $N(x)$ is the cumulative normal distribution function of:

$$d_1 = \frac{ln(\frac{S}{K}) + (\omega + \frac{\sigma_a^2}{2})T}{\sigma_a\sqrt{T}},$$

$$d_2 = \frac{ln(\frac{S}{K}) - (\omega + \frac{\sigma_a^2}{2})T}{\sigma_a \sqrt{T}},$$

which can be simplified to

$$d_2 = d_1 - \sigma_a \sqrt{T}.$$

The adjusted volatility and dividend yield are given as follows:

$$\sigma_a = \frac{\sigma}{\sqrt{3}}$$

and

$$\omega = \frac{1}{2}\left(r - \lambda - \frac{\sigma^2}{6}\right).$$

Here σ_a is the adjusted volatility, σ is the observed volatility, r is the risk free rate of interest, K is the strike price or exercise price, S is the underlying price of the asset and λ is the dividend yield. The algorithm above can easily be implemented in Matlab to estimate the price of Asian call (put) options.

3.3. Lookback Option

Here we shall consider the Monte Carlo approach for pricing Lookback option under the CEV process [5]. The stock price $S(t)$ follows the following diffusion process under risk neutral measure,

$$dS(t) = \mu S(t)dt + \sigma S^{\frac{\alpha}{2}}(t)dW(t), \tag{25}$$

where μ, σ and α are constants and $\{W(t), t > 0\}$ is a standard Brownian motion of Wiener process. If $\alpha = 2$, we are back to the standard lognormal diffusion case in (1) and we have a very simple closed form expression for the lookback option prices. Now,we obtain the estimated call and put option prices for lookback option as follows: we simulate the asset price for N days, then we have for the antithetic techniques

$$S_+^i[(j+1)\Delta t] = S(j\Delta t)\exp\left[(r - \frac{\sigma^2}{2})\Delta t + \sigma\sqrt{\Delta t}Z_j\right] \tag{26}$$

$$S_-^i[(j+1)\Delta t] = S(j\Delta t)\exp\left[(r - \frac{\sigma^2}{2})\Delta t - \sigma\sqrt{\Delta t}Z_j\right] \tag{27}$$

for $j = 0, 1, 2, ..., N-1,\ \ i = 1, 2, ..., M$ and where S_+ and S_- are positive and negative antithetic values of the stock price respectively. Next we obtain the max(min) asset price for the put(call) reached during the life of the option for each of the equation in (26) and (27). Then the estimates of the ith simulation for the call options are given by

$$C^i_{+f} = \max[S_+(t_N) - m_+, 0], \tag{28}$$

$$C^i_{-f} = \max[S_-(t_N) - m_+, 0], \tag{29}$$

where $m_+ = \min S_+(t_j)$ and $m_- = \min S_-(t_j)$, for $j = 1, 2, ..., N$. The values $S_+(t_N)$ and $S_-(t_N)$ are the stock prices at maturity time T. The estimates of the put options are given by

$$P^i_{+f} = \max[M_+ - S_+(t_N), 0], \tag{30}$$

$$P^i_{-f} = \max[M_- - S_-(t_N), 0], \tag{31}$$

where $M_+ = \max S_+(t_j)$ and $M_- = \max S_-(t_j)$, for $j = 1, 2, ..., N$. We repeat the procedure for M simulated sample paths. The respective estimated call and put option prices are

$$C_f = \frac{e^{-rT}}{2M}\left[\sum_{i=1}^{M} C^i_{+f} + \sum_{i=1}^{M} C^i_{-f}\right], \tag{32}$$

$$P_f = \frac{e^{-rT}}{2M}\left[\sum_{i=1}^{M} P^i_{+f} + \sum_{i=1}^{M} P^i_{-f}\right]. \tag{33}$$

Hence the case $\alpha = 2$ forms a very natural control variate for the problem. The algorithm can easily be implemented in Matlab to estimate the prices of the floating strike price lookback options.

4. Numerical Examples

Now, we present some numerical examples as follows:

Example 1. We consider the performance of the Monte Carlo method for geometric average Asian options with

$$S_0 = 100,\ r = 0.05,\ \sigma = 0.2,\ T = 1,\ n = 10000.$$

The results obtained are shown in Table 1 below.

Example 2. We consider the performance of the Monte Carlo method for lookback options with

$$S_0 = 100,\ r = 0.1,\ \sigma = 0.25,\ T = 0.5.$$

The results obtained are shown in Table 2 and Table 3 below.

4.1. Discussion of Results

Table 1 shows the limit of the Monte Carlo method for pricing Asian call options. As K increases, although the standard error decreases, the simulations become less efficient for the reason that the estimates go down much faster than the corresponding standard errors. On the other hand, when the time step number gets larger, the standard error increases. Tables 2 and 3 show the performance of Monte Carlo for pricing Lookback call and put options respectively, the resulting values are extremely accurate and that they are consistent with the accurate results. Monte Carlo method is useful in pricing path dependent options and is becoming more appealing and gaining popularity in derivative pricing.

5. Conclusion

We have at our disposal one numerical method for pricing some path dependent options. In general, numerical method has its advantages and disadvantages of use. The Monte Carlo method works very well for pricing path dependent options especially Asian Options, approximates every arbitrary exotic options, it is flexible in handling varying and even high dimensional financial problems. Moreover, despite its significant progress, early exercise remain problematic for Monte Carlo method. This method does poor job in pricing continuously monitored lookbacks even under the lognormal assumption due to the discretization of the stochastic differential equation and simulate it at discrete points which affects the information about the part of the path between observation dates.

Finally, when pricing options the important thing is to choose the correct numerical method from the wide array of methods available.

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Appendices

K	**M**	**Black-Scholes**	**G. Asian Call**	**M.C. Method**	**Standard error**
90	10	16.6994	12.2398	12.5883	0.1020
	20		12.2764	12.6718	0.1037
	50		12.3009	12.0587	0.1050
	100		12.3092	12.6495	0.1050
	200		12.3134	12.6213	0.1044
	500		12.3160	12.8864	0.1056
100	10	10.4506	5.4294	5.7874	0.0798
	20		5.4856	5.7047	0.0782
	50		5.5217	5.7255	0.0786
	100		5.5341	5.6107	0.0780
	200		5.5404	5.9079	0.0815
	500		5.5443	5.8107	0.0799
110	10	6.0401	1.7500	1.8700	0.0458
	20		1.7952	1.9970	0.0487
	50		1.8243	2.0545	0.0503
	100		1.8344	1.9811	0.0486
	200		1.8395	1.9807	0.0485
	500		1.8426	2.0304	0.0493

Table 1: Monte Carlo Method for Geometric Average Asian Call Options

$K = S_{\min}$	**Accurate Lbk Call Price**	**M.C. Method**	**Standard Error**
90	18.1821	18.1821	0.0001
95	16.2960	16.2960	0.0001
100	15.6359	15.6359	0.0001

Table 2: Lookback Call Option

$K = S_{\max}$	**Accurate Lbk Call Price**	**M.C. Method**	**Standard Error**
100	12.2827	12.2827	0.0001
105	12.7039	12.7039	0.0001
110	13.9610	13.9610	0.0001

Table 3: Lookback Put Option

The above results can be obtained using Matlab codes.