International Journal of Applied Mathematics
Volume 25 No. 6 2012, 845-855

EXTENSION OF SOME POLYNOMIAL INEQUALITIES TO THE POLAR DERIVATIVE

N.A. Rather[1], Suhail Gulzar[2 §]
[1,2] Department of Mathematics
University of Kashmir
Srinagar, Hazratbal, 190006, INDIA
[1]e-mail: dr.narather@gmail.com
[2]e-mail: sgmattoo@gmail.com

Abstract: Let $P(z)$ be a polynomial of degree n having all zeros in $|z| \leq k$, where $k \leq 1$, then it was proved by Dewan *et al* [5] that for every real or complex number α with $|\alpha| \geq k$ and each $r \geq 0$

$$n(|\alpha|-k)\left\{\int_0^{2\pi}\left|P\left(e^{i\theta}\right)\right|^r d\theta\right\}^{\frac{1}{r}} \leq \left\{\int_0^{2\pi}\left|1+ke^{i\theta}\right|^r d\theta\right\}^{\frac{1}{r}} \max_{|z|=1}|D_\alpha P(z)|.$$

In this paper, we extend the above inequality to the class of polynomials $P(z) = a_n z^n + \sum_{\nu=\mu}^{n} a_{n-\nu} z^{n-\nu}$, $1 \leq \mu \leq n$, having all its zeros in $|z| \leq k$ where $k \leq 1$ and thereby obtain generalizations and refinements of above and many other known results.

AMS Subject Classification: 30A10, 30C10, 30E10
Key Words: polynomials, polar derivatives, integral mean estimates

1. Introduction and Statement of Results

Let $P(z)$ be a polynomial of degree n. It was shown by Turán [10] that if $P(z)$ has all its zeros in $|z| \leq 1$, then

$$n\max_{|z|=1}|P(z)| \leq 2\max_{|z|=1}\left|P'(z)\right|. \tag{1.1}$$

Inequality (1.1) is the best possible with equality holding for $P(z) = \alpha z^n + \beta$,

Received: November 9, 2012
§Correspondence author