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EVALUATION OF THE BETA FUNCTION

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Abstract: The Beta function $B(x,y)$ is defined for negative integer values of x and y. It is proved that

$$B(n+1/2,-n) = \frac{(-1)^n(2n)!}{2^{2n-1}(n!)^2}\ln 2 + \frac{(-1)^n\phi(n)(2n)!}{2^{2n}(n!)^2}$$

for $n = 1, 2, \ldots$, where

$$\phi(n) = \sum_{i=1}^{n} 1/n$$

and

$$\begin{aligned} B(n+3/2,-1) &= \sum_{i=1}^{n}\frac{2n+1}{2i-1} - (2n+1)\ln 2 - \frac{2n+1}{2}, \\ B(-n+1/2,-1) &= -\sum_{i=1}^{n+1}\frac{2n+1}{2i-1} + (2n+1)\ln 2 + \frac{2n+1}{2} \end{aligned}$$

for $n = 1, 2, \ldots$.

Further results are also given.

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The Beta function $B(x,y)$, see Sneddon [5], is usually defined by the integral

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\,dt, \tag{1}$$

for $x,y > 0$ and more generally the function

$$\frac{\partial^{p+q}}{\partial x^p \partial y^q} B(x,y) = B_{p,q}(x,y)$$

is defined by the integral

$$B_{p,q}(x,y) = \int_0^1 t^{x-1}\ln^p t(1-t)^{y-1}\ln^q(1-t)\,dt,$$

for $x,y > 0$ and $p,q = 0,1,2,\ldots$.

It follows from equation (1) that

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

and this equation is then used to define $B(x,y)$ for $x,y < 0$ and $x,y \neq -1,-2,\ldots$.

More generally it was proved in [3] that the neutrix limit

$$B_{p,q}(x,y) = \mathrm{N}\!-\!\lim_{\epsilon\to 0} \int_\epsilon^{1-\epsilon} t^{x-1}\ln^p t(1-t)^{y-1}\ln^q(1-t)\,dt, \tag{2}$$

exists for all x,y and $p,q = 0,1,2,\ldots$, where N is the neutrix, see van der Corput [1], having domain $N' = (0,x) \quad (0 < x < 1/2)$ and range N" the real numbers, with the negligible functions finite linear sums of the functions

$$\epsilon^\lambda \ln^{r-1}\epsilon, \quad \ln^r \epsilon \qquad (\lambda < 0, \quad r = 1,2,\ldots)$$

and all functions which converge to zero in the normal sense as ϵ tends to zero. Equation (2) was then used to define $B_{p,q}(x,y)$ for all x,y and $p,q = 0,1,2,\ldots$.

Note that if $x > 0$, we could write

$$B_{p,q}(x,y) = \mathrm{N}\!-\!\lim_{\epsilon\to 0} \int_0^{1-\epsilon} t^{x-1}\ln^p t(1-t)^{y-1}\ln^q(1-t)\,dt,$$

and if $y > 0$, we could write

$$B_{p,q}(x,y) = \mathrm{N}\!-\!\lim_{\epsilon\to 0} \int_\epsilon^1 t^{x-1} \ln^p t(1-t)^{y-1} \ln^q(1-t)\, dt.$$

The following theorem was proved in [4]:

Theorem 1.

$$B_{p,q}(x,y) = B_{q,p}(y,x)$$

for all x, y *and* $p, q = 0, 1, 2, \ldots$.

We now prove

Theorem 2.

$$B(1/2, 0) = \ln 4 \tag{3}$$

and

$$B(n+1/2, 0) = -\sum_{i=1}^n \frac{2}{2i-1} + \ln 4, \tag{4}$$

$$B(-n+1/2, 0) = -\sum_{i=1}^n \frac{2}{2i-1} + \ln 4 \tag{5}$$

for $n = 1, 2, \ldots$.

Proof. We first prove equation (3). Putting $t = u^2$, we have

$$\int_0^{1-\epsilon} t^{-1/2}(1-t)^{-1}\, dt = 2\int_0^{\sqrt{1-\epsilon}} (1-u^2)^{-1}\, du = \ln \frac{1+\sqrt{1-\epsilon}}{1-\sqrt{1-\epsilon}}. \tag{6}$$

Now

$$1 - \sqrt{1-\epsilon} = \frac{\epsilon[1+O(\epsilon)]}{2}$$

and so

$$\mathrm{N}\!-\!\lim_{\epsilon\to 0} \ln(1-\sqrt{1-\epsilon}) = -\ln 2.$$

It now follows from equation (6) that

$$B(1/2, 0) = \mathrm{N}\!-\!\lim_{\epsilon\to 0} \int_0^{1-\epsilon} t^{-1/2}(1-t)^{-1}\, dt = \ln 4,$$

proving equation (3).

Next, we have

$$\begin{aligned}\int_0^{1-\epsilon} t^{n-3/2}(1-t)^{-1}\,dt &= \int_0^{1-\epsilon} t^{n-1/2}\Big(\frac{1}{t}+\frac{1}{1-t}\Big)\,dt \\ &= \frac{2(1-\epsilon)^{n-1/2}}{2n-1} + \int_0^{1-\epsilon}\frac{t^{n-1/2}}{1-t}\,dt\end{aligned}$$

and it follows that

$$\begin{aligned}\mathop{\mathrm{N-lim}}_{\epsilon\to 0}\int_0^{1-\epsilon} t^{n-3/2}(1-t)^{-1}\,dt\,dt &= B(n-1/2,0) \\ &= \frac{2}{2n-1} + B(n+1/2,0)\end{aligned}$$

for $n = 1, 2, \ldots$. Hence

$$B(1/2,0) = \sum_{i=1}^{n}\frac{2}{2i-1} + B(n+1/2,0) = \ln 4$$

and equation (4) follows for $n = 1, 2, \ldots$.

Next, we have

$$\begin{aligned}\int_\epsilon^{1-\epsilon} t^{-n-1/2}(1-t)^{-1}\,dt &= \int_\epsilon^{1-\epsilon} t^{-n+1/2}\Big(\frac{1}{t}+\frac{1}{1-t}\Big)\,dt \\ &= -\frac{2(1-\epsilon)^{-n+1/2}}{2n-1} + \frac{2\epsilon^{-n+1/2}}{2n-1} + \int_\epsilon^{1-\epsilon}\frac{t^{-n+1/2}}{1-t}\,dt\end{aligned}$$

and it follows that

$$\begin{aligned}\mathop{\mathrm{N-lim}}_{\epsilon\to 0}\int_\epsilon^{1-\epsilon} t^{-n+1/2}\Big(\frac{1}{t}+\frac{1}{1-t}\Big)\,dt &= B(-n+1/2,0) \\ &= -\frac{2}{2n-1} + B(-n+3/2,0)\end{aligned}$$

for $n = 1, 2, \ldots$. Hence

$$\begin{aligned}B(-n+1/2,0) &= -\sum_{i=1}^{n}\frac{2}{2i-1} + B(1/2,0) \\ &= -\sum_{i=1}^{n}\frac{2}{2i-1} + \ln 4,\end{aligned}$$

proving equation (5) for $n = 1, 2, \ldots$. □

Theorem 3.

$$B(n+1/2,-n) = \frac{(-1)^n(2n)!}{2^{2n-1}(n!)^2}\ln 2 + \frac{(-1)^n\phi(n)(2n)!}{2^{2n}(n!)^2} \tag{7}$$

for $n = 1, 2, \ldots$, *where*

$$\phi(n) = \sum_{i=1}^{n} 1/n$$

and

$$B(n+3/2,-1) = \sum_{i=1}^{n}\frac{2n+1}{2i-1} - (2n+1)\ln 2 - \frac{2n+1}{2}, \tag{8}$$

$$B(-n+1/2,-1) = -\sum_{i=1}^{n+1}\frac{2n+1}{2i-1} + (2n+1)\ln 2 + \frac{2n+1}{2} \tag{9}$$

for $n = 1, 2, \ldots$.

In general

$$\begin{aligned} B(n+r-1/2,-n+1) &= \frac{(-1)^n(2n+2r)!r!}{2^{2n-1}(2n+2r-1)n!(n+r)!(2r)!} \\ &\quad - \frac{2n}{2n+2r-1}B(n+r+1/2,-n), \end{aligned} \tag{10}$$

for $r = 0, 1, 2, \ldots$ *and* $n = 1, 2, \ldots$.

Proof. We have

$$\begin{aligned} \int_0^{1-\epsilon} t^{n-3/2}(1-t)^{-n}\,dt &= \frac{2}{2n-1}\int_0^{1-\epsilon}(1-t)^{-n}\,dt^{n-1/2} \\ = \frac{2}{2n-1}(1-\epsilon)^{n-1/2}\epsilon^{-n} &- \frac{2n}{2n-1}\int_0^{1-\epsilon} t^{n-1/2}(1-t)^{-n-1}\,dt \end{aligned}$$

and so

$$\begin{aligned} \mathop{\mathrm{N-lim}}_{\epsilon\to 0}\int_0^{1-\epsilon} t^{n-3/2}(1-t)^{-n}\,dt &= B(n-1/2,-n+1) \\ &= \frac{(-1)^n(2n)!}{2^{2n-1}(2n-1)(n!)^2} - \frac{2n}{2n-1}B(n+1/2,-n), \end{aligned} \tag{11}$$

since

$$\mathop{\mathrm{N-lim}}_{\epsilon\to 0}(1-\epsilon)^{n-1/2}\epsilon^{-n} = \frac{(-1)^n(2n)!}{2^{2n}(n!)^2},$$

for $n = 1, 2, \ldots$.

In particular, when $n = 1$, it follows from equation (11) that

$$B(1/2, 0) = -1 - 2B(3/2, -1) = \ln 4$$

on using equation (3) and so equation (7) is true when $n = 1$.

Now suppose that equation (7) holds for some n. Then it follows from equation (11) that

$$\begin{aligned}\frac{2n+2}{2n+1}B(n+3/2, -n-1) &= \frac{(-1)^{n+1}(2n+2)!}{2^{2n+1}(2n+1)[(n+1)!]^2} - \frac{(-1)^n(2n)!}{2^{2n-1}(n!)^2}\ln 2 \\ &\quad - \frac{(-1)^n\phi(n)(2n)!}{2^{2n}(n!)^2}\end{aligned}$$

and so

$$\begin{aligned}B(n+3/2, -n-1) &= \frac{(-1)^{n+1}(2n+2)!}{2^{2n-1}(2n+2)^2(n!)^2}\ln 2 + \frac{(-1)^{n+1}\phi(n)(2n+2)!}{2^{2n}(2n+2)^2(n!)^2} + \\ &\quad + \frac{(-1)^{n+1}(2n+2)!}{2^{2n+2}(n+1)[(n+1)!]^2} \\ &= \frac{(-1)^{n+1}(2n+2)!}{2^{2n+1}[(n+1)!]^2}\ln 2 + \frac{(-1)^{n+1}\phi(n)(2n+2)!}{2^{2n+2}[(n+1)!]^2} + \\ &\quad + \frac{(-1)^{n+1}(2n+2)!}{2^{2n+2}(n+1)[(n+1)!]^2} \\ &= \frac{(-1)^{n+1}(2n+2)!}{2^{2n+1}[(n+1)!]^2}\ln 2 + \frac{(-1)^{n+1}\phi(n+1)(2n+2)!}{2^{2n+2}[(n+1)!]^2}\end{aligned}$$

proving that equation (7) is true for $n+1$. Equation (7) now follows by induction for $n = 1, 2, \ldots$.

Next, we have

$$\begin{aligned}\int_0^{1-\epsilon} t^{n-1/2}(1-t)^{-1}\,dt &= \frac{2}{2n+1}\int_0^{1-\epsilon}(1-t)^{-1}\,dt^{n+1/2} \\ = \frac{2}{2n+1}(1-\epsilon)^{n+1/2}\epsilon^{-1} &- \frac{2}{2n+1}\int_0^{1-\epsilon} t^{n+1/2}(1-t)^{-2}\,dt\end{aligned}$$

and so

$$\begin{aligned}\mathrm{N}\!-\!\lim_{\epsilon\to 0}\int_0^{1-\epsilon} t^{n-1/2}(1-t)^{-1}\,dt &= B(n+1/2, 0) = -\sum_{i=1}^{n}\frac{2}{2i-1} + \ln 4 \\ &= -1 - \frac{2}{2n+1}B(n+3/2, -1),\end{aligned}$$

since

$$\mathop{\mathrm{N-lim}}_{\epsilon\to 0}(1-\epsilon)^{n+1/2}\epsilon^{-1} = -\frac{2n+1}{2}.$$

Equation (8) follows for $n = 1, 2, \ldots$.

To prove equation (9), we have

$$\begin{aligned}
\int_\epsilon^{1-\epsilon} t^{-n-3/2}(1-t)^{-1}\,dt &= -\frac{2}{2n+1}\int_\epsilon^{1-\epsilon}(1-t)^{-1}\,dt^{-n-1/2} \\
&= -\frac{2}{2n+1}(1-\epsilon)^{-n-1/2}\epsilon^{-1} \\
&\qquad +\frac{2}{2n+1}(1-\epsilon)^{-1}\epsilon^{-n-1/2} \\
&\qquad +\frac{2}{2n+1}\int_\epsilon^{1-\epsilon} t^{-n-1/2}(1-t)^{-2}\,dt
\end{aligned}$$

and so

$$\begin{aligned}
\mathop{\mathrm{N-lim}}_{\epsilon\to 0}\int_\epsilon^{1-\epsilon} t^{-n-3/2}(1-t)^{-1}\,dt &= B(-n-1/2, 0) \\
&= -\sum_{i=1}^{n+1}\frac{2}{2i-1} + \ln 4 \\
&= -1 + \frac{2}{2n+1}B(-n+1/2, -1),
\end{aligned}$$

on using equation (5), since

$$\mathop{\mathrm{N-lim}}_{\epsilon\to 0}(1-\epsilon)^{-n-1/2}\epsilon^{-1} = \frac{2n+1}{2}.$$

Equation (9) follows for $n = 1, 2, \ldots$.

In general, we have

$$\begin{aligned}
\int_0^{1-\epsilon} t^{n+r-3/2}(1-t)^{-n}\,dt &= \frac{2}{2n+2r-1}\int_0^{1-\epsilon}(1-t)^{-n}\,dt^{n+r-1/2} \\
= \frac{2}{2n+2r-1}(1-\epsilon)^{n+r-1/2}\epsilon^{-n} &- \frac{2n}{2n+2r-1}\int_0^{1-\epsilon} t^{n+r-1/2}(1-t)^{-n-1}\,dt
\end{aligned}$$

and so

$$\begin{aligned}\mathrm{N}\!-\!\lim_{\epsilon\to 0}\int_0^{1-\epsilon} t^{n+r-3/2}(1-t)^{-n}\,dt &= B(n+r-1/2,-n+1)\\ &= \frac{(-1)^n(2n+2r)!r!}{2^{2n-1}(2n+2r-1)n!(n+r)!(2r)!}\\ &\quad -\frac{2n}{2n+2r-1}B(n+r+1/2,-n),\end{aligned} \tag{12}$$

since

$$\mathrm{N}\!-\!\lim_{\epsilon\to 0}(1-\epsilon)^{n+r-1/2}\epsilon^{-n} = \frac{(-1)^n(2n+2r)!r!}{2^{2n}n!(n+r)!(2r)!},$$

for $r = 0, 1, 2, \ldots$ and $n = 1, 2, \ldots$. Equation (10) follows. □

Theorem 4.

$$B_{0,1}(-1/2,1) = -4\ln 2 \tag{13}$$

and

$$B_{0,1}(n-1/2,1) = -\frac{4}{2n-1}\left[\sum_{i=1}^{n}\frac{1}{2i-1}-\ln 2\right], \tag{14}$$

$$B_{0,1}(-n-1/2,1) = \frac{4}{2n+1}\left[\sum_{i=1}^{n}\frac{1}{2i-1}-\ln 2\right] \tag{15}$$

for $n = 1, 2, \ldots$.

More generally,

$$B_{0,1}(-1/2,r+1) = -4\ln 2+\sum_{j=1}^{r}\binom{r}{j}\frac{4(-1)^j}{2j-1}\left[-\sum_{i=1}^{j}(2i-1)^{-1}+\ln 2\right] \tag{16}$$

for $r = 1, 2, \ldots,$

$$B_{0,1}(n-1/2,r+1) = \sum_{j=0}^{r}\binom{r}{j}\frac{4(-1)^j}{2n+2j-1}\left[-\sum_{i=1}^{n+j}(2i-1)^{-1}+\ln 2\right] \tag{17}$$

for $r, n = 1, 2, \ldots,$

$$B_{0,1}(-n-1/2,r+1) = \sum_{j=0}^{r}\binom{r}{j}\frac{4(-1)^j}{2n-2j+1}\left[\sum_{i=1}^{n-j}(2i-1)^{-1}-\ln 2\right] \tag{18}$$

for $n = 1, 2, \ldots$ *and* $r = 1, 2, \ldots, n-1$,

$$B_{0,1}(-n-1/2, n+1) = (-1)^{n+1} 4 \ln 2 + \sum_{j=0}^{n-1} \binom{r}{j} \frac{4(-1)^j}{2n-2j+1} \left[\sum_{i=1}^{n-j} (2i-1)^{-1} - \ln 2 \right] \tag{19}$$

for $n = 1, 2, \ldots$ *and*

$$\begin{aligned} B_{0,1}(-n-1/2, r+1) &= \sum_{j=0}^{n-1} \binom{r}{j} \frac{4(-1)^j}{2n-2j+1} \left[\sum_{i=1}^{n-j} (2i-1)^{-1} - \ln 2 \right] \\ &\quad - \binom{r}{n} (-1)^n 4 \ln 2 \\ &\quad + \sum_{j=n+1}^{r} \binom{r}{j} \frac{4(-1)^j}{2j-2n-1} \left[\ln 2 - \sum_{i=1}^{j-n} (2i-1)^{-1} \right] \end{aligned} \tag{20}$$

for $n = 1, 2, \ldots$ *and* $r = n+1, n+2, \ldots$.

Proof. We have

$$\begin{aligned} \int_0^{1-\epsilon} t^{n-1/2} (1-t)^{-1} \, dt &= -\int_0^{1-\epsilon} t^{n-1/2} \, d \ln(1-t) \\ &= -(1-\epsilon)^{n-1/2} \ln \epsilon + \frac{2n-1}{2} \int_0^{1-\epsilon} t^{n-3/2} \ln(1-t) \, dt. \end{aligned}$$

Hence

$$\begin{aligned} \mathrm{N}\!-\!\lim_{\epsilon \to 0} \int_0^{1-\epsilon} t^{n-1/2} (1-t)^{-1} \, dt &= \frac{2n-1}{2} B_{0,1}(n-1/2, 1) \\ &= B(n+1/2, 0) \\ &= -\sum_{i=1}^{n} \frac{2}{2i-1} + 2 \ln 2 \end{aligned}$$

on using equation (4) and equation (14) follows for $n = 1, 2, \ldots$.

Next, we have

$$\begin{aligned} \int_\epsilon^{1-\epsilon} t^{-n-1/2} (1-t)^{-1} \, dt &= -\int_\epsilon^{1-\epsilon} t^{-n-1/2} \, d \ln(1-t) \\ &= -(1-\epsilon)^{-n-1/2} \ln \epsilon + \epsilon^{-n-1/2} \ln(1-\epsilon) \\ &\quad - \frac{2n+1}{2} \int_\epsilon^{1-\epsilon} t^{-n-3/2} \ln(1-t) \, dt. \end{aligned}$$

It follows that

$$\begin{aligned}
\mathrm{N}\!-\!\lim_{\epsilon\to 0}\int_{\epsilon}^{1-\epsilon} t^{-n-1/2}(1-t)^{-1}\,dt &= -\frac{2n+1}{2}B_{0,1}(-n-1/2,1) \\
&= B(-n+1/2,0) \\
&= -\sum_{i=1}^{n}\frac{2}{2i-1}+2\ln 2 \qquad (21)
\end{aligned}$$

on using equation (5) and equation (15) follows for $n = 1, 2, \ldots$.

In the particular case $n = 0$, equation (21) is replaced by the equation

$$-\frac{1}{2}B_{0,1}(-1/2,1) = B(1/2,0) = 2\ln 2$$

on using equation (3) and equation (13) follows.

To prove equation (16), we have

$$\int_{\epsilon}^{1} t^{-3/2}(1-t)^{r}\ln(1-t)\,dt = \sum_{j=0}^{r}\binom{r}{j}(-1)^{j}\int_{\epsilon}^{1} t^{j-3/2}\ln(1-t)\,dt$$

and it follows that

$$\begin{aligned}
B_{0,1}(-1/2,r+1) &= \mathrm{N}\!-\!\lim_{\epsilon\to 0}\int_{\epsilon}^{1} t^{-3/2}(1-t)^{r}\ln(1-t)\,dt \\
&= \sum_{j=0}^{r}\binom{r}{j}(-1)^{j}B_{0,1}(j-1/2,1) \\
&= -4\ln 2+\sum_{j=1}^{r}\binom{r}{j}\frac{4(-1)^{j}}{2j-1}\left[-\sum_{i=1}^{j}\frac{1}{2i-1}+\ln 2\right]
\end{aligned}$$

on using equations (13) and (14). Equation (16) follows for $r = 1, 2, \ldots$.

More generally, we have

$$\begin{aligned}
B_{0,1}(n-1/2,r+1) &= \int_{0}^{1} t^{n-3/2}(1-t)^{r}\ln(1-t)\,dt \\
&= \sum_{j=0}^{r}\binom{r}{j}(-1)^{j}\int_{0}^{1} t^{n+j-3/2}\ln(1-t)\,dt \\
&= \sum_{j=0}^{r}\binom{r}{j}(-1)^{j}B_{0,1}(n+j-1/2,1) \\
&= \sum_{j=0}^{r}\binom{r}{j}\frac{4(-1)^{j}}{2n+2j-1}\left[-\sum_{i=1}^{n+j}(2i-1)^{-1}+\ln 2\right]
\end{aligned}$$

on using equation (14) and equation (17) follows for $r, n = 1, 2, \ldots$.

Next, we have

$$\int_{\epsilon}^{1} t^{-n-3/2}(1-t)^r \ln(1-t)\,dt = \sum_{j=0}^{r} \binom{r}{j}(-1)^j \int_{\epsilon}^{1} t^{-n+j-3/2} \ln(1-t)\,dt$$

and it follows that

$$\begin{aligned} B_{0,1}(-n-1/2, r+1) &= \mathrm{N}\!-\!\lim_{\epsilon\to 0} \int_{\epsilon}^{1} t^{-n-3/2}(1-t)^r \ln(1-t)\,dt \\ &= \sum_{j=0}^{r} \binom{r}{j}(-1)^j B_{0,1}(-n+j-1/2, 1) \qquad (22) \\ &= \sum_{j=0}^{r} \binom{r}{j} \frac{4(-1)^j}{2n-2j+1} \left[\sum_{i=1}^{n-j} (2i-1)^{-1} - \ln 2 \right] \end{aligned}$$

on using equation (15). Equation (18) follows for $n = 1, 2, \ldots$ and $r = 1, 2, \ldots, n-1$.

When $r = n$, we have from equation (22)

$$B_{0,1}(-n-1/2, n+1) = \sum_{j=0}^{n-1} \binom{n}{j}(-1)^j B_{0,1}(-n+j-1/2, 1) + (-1)^n B_{0,1}(-1/2, 1)$$

and equation (19) follows from equations (13) and (15).

When $r > n$, we have from equation (22)

$$\begin{aligned} B_{0,1}(-n-1/2, r+1) &= \sum_{j=0}^{n-1} \binom{r}{j}(-1)^j B_{0,1}(-n+j-1/2, 1) \\ &\quad + \binom{r}{n}(-1)^n B_{0,1}(-1/2, 1) + \sum_{j=n+1}^{r} \binom{r}{j}(-1)^j B_{0,1}(-n+j-1/2, 1) \end{aligned}$$

and equation (20) follows from equations (13), (14) and (15). □

For further results, see [2].

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