

**A DERIVATION OF THE HIGGS MASS
TO BE 123 GEV AND A NOTE ON
THE WEINBERG ANGLE**

Gregory L. Light

Department of Finance

Providence College

Providence, Rhode Island, 02918, USA

Abstract: We derive the Higgs mass to be 123 GeV; we also show that the Higgs field would also impart a rest mass to the photon despite the well-known linear transformation of (A, b₃) to (photon, Z).

AMS Subject Classification: 81V22, 81V15, 81V10, 81T10

Key Words: Weinberg angle, Higgs mass, spontaneous symmetry breaking

1. Introduction

We give an annotated analysis of the exposition of the Standard Electroweak Model by Quigg ([4], in particular, pp. 121-127). We will derive the rest energy of the Higgs particle to be 123 GeV, in near perfect agreement with the observed 125 GeV by the highly publicized LHC finding on July 4, 2012 (cf. [2] for pursuing a model to derive this result). However, we contend that the role played by the Higgs is not that of endowing rest masses to particles, rather it serves as the source of the weak nuclear force by generating the W^\pm and Z_0 bosons (cf. [3]). In this connection, we will show that the Higgs boson in the electroweak integration imparts rest masses not only to W^\pm and Z_0 but also to photons despite the linear transformation (known as the Weinberg angle) of the Maxwell electromagnetic 4-potential A_μ^M along with the postulated $b_\mu^{(3)}$ into a photon field $A_{0,\mu}$ along with $Z_{0,\mu}$ (by our extensive literature research, no similar

calculations to ours existing in the literature; for using the Higgs field, without the construct of spontaneous symmetry breaking, to yield rest masses to W^\pm and Z_0 but not to photons, cf. [1]). Since photons do not possess rest masses but A_0 does, using A_0 as the gauge boson to explain electromagnetism is tantamount to the statement that photons are not the gauge boson of electromagnetism and consequently the electroweak theory breaks down (cf. [4], p.1: "Moreover, we have reason to believe that the electroweak theory is imperfect and that new symmetries or new dynamical principles are required to make it fully robust."). If $U(1) \times SU(2)$ fails to be unified, then the entire Standard Model, $U(1) \times SU(2) \times SU(3)$, built upon the idea of unification of forces loses its theoretical ground (see for our explanation of the strong nuclear force in [3]).

We display below the formalism used in [4] for our presentation in Section 2, and in Section 3 we will draw a conclusion.

$$L = L_{gauge} + L_{leptons},$$

$$L_{gauge} = -\frac{1}{4}F_{\mu\nu}^l F^{l\mu\nu} - \frac{1}{4}f_{\mu\nu} f^{\mu\nu}, \quad l = 1, 2, 3,$$

$$\mu, \nu \in \{t, x, y, z\}, \text{ repeated upper/lower indices summed,}$$

$$F_{\mu\nu}^l = \partial_\nu b_\mu^l - \partial_\mu b_\nu^l + g\epsilon_{jkl} b_\mu^j b_\nu^k,$$

$$f_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu,$$

$$L_{leptons} = \bar{R}i\gamma^\mu \left(\partial_\mu + \frac{ig'}{2}A_\mu Y \right) R$$

$$+ \bar{L}i\gamma^\mu \left(\partial_\mu + \frac{ig'}{2}A_\mu Y + \frac{ig}{2}\tau \cdot \mathbf{b}_\mu \right) L, \quad (1.1)$$

$$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix},$$

$$L_{scalar} = (D^\mu \phi)^\dagger (D_\mu \phi) - V(\phi^\dagger \phi),$$

$$D_\mu \equiv \partial_\mu + \frac{ig'}{2}A_\mu Y + \frac{ig}{2}\tau \cdot \mathbf{b}_\mu, \text{ and}$$

$$V(\phi^\dagger \phi) = \mu^2 (\phi^\dagger \phi) + |\lambda| (\phi^\dagger \phi)^2.$$

For communication to the general audience in Applied Mathematics, we will however make certain changes to some of the above symbols.

2. Analysis

The linchpin of the Standard Model is the Higgs particle, which purports to be the source of rest masses of all particles except the photon. In the following we will, as based on [4], (1) give a derivation of the mass of the Higgs particle to be 123 GeV, and (2) show that the abstract rotation by the Weinberg angle fails to stop the Higgs particle to assign a rest mass to the photon so that photons as from the electroweak integration would nevertheless possess rest masses.

2.1. The Higgs Mass

Set

$$z \equiv \phi_+ + \phi_0 i \equiv \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix},$$

where we interpret:

$\hbar\phi_+$ = the kinetic energy (KE) density of the Higgs field associated with a Higgs particle, which would carry a positive electric charge (Coulomb), and

$\hbar\phi_0$ = the rest energy (RE) density of this Higgs field.

Then

$$m^6 \left[(\hbar\phi_+)^2 + (\hbar\phi_0)^2 \right] = KE^2 + RE^2 = E^2$$

by the mass-shell equation of Einstein's Special Relativity. That is, the fact that

$$z^* z = |z|^2 = (\phi_+, \phi_0) \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix}$$

must not be mistaken for the Euclidean norm-squared of a vector in \mathbb{R}^3 . Now since $z = \begin{pmatrix} \phi_+ > 0 \\ \phi_0 > 0 \end{pmatrix}$ would represent a particle that possesses an electric charge if and only if the particle carries a kinetic energy (to a frame), i.e., in motion (to the frame), one must conclude that $z = \begin{pmatrix} \phi_+ > 0 \\ \phi_0 > 0 \end{pmatrix}$ would have to be a magnetic monopole. As such, since no magnetic monopoles have been found, one models

$$z = \begin{pmatrix} 0 \\ \phi_0 > 0 \end{pmatrix};$$

then this is a particle that has no motions to any frame, implying that z attaches itself to some particle p_α so that z has no motion to p_α ; moreover, since p_α can move relative to any frame, z must lose its identity once it gets attached to p_α , implying that z must become an inherent property of p_α .

Having settled $z = \begin{pmatrix} 0 \\ \phi_0 > 0 \end{pmatrix}$, the Standard Model has the following Lagrangian

$$\begin{aligned} L_{scalar} &= (D^\mu z)^* (D_\mu z) - V(z^* z), \text{ where} \\ D_\mu &\equiv i\hbar \cdot \left(\frac{\partial}{\partial x_\mu} + \frac{ig_E}{2} A_\mu^M Y + \frac{ig_W}{2} \tau \cdot \mathbf{b}_\mu \right), \end{aligned} \quad (2.1)$$

$g_E \equiv$ a unit-free real number > 0 , measuring the probability of an interaction between the electron contained in the Maxwell 4-*potential* \mathbf{A}^M and the electron that is contained in the wavefunctions L or R in $L_{leptons}$ (Equation (1.1)),

$Y \equiv 2(Q - I_3) = 2(-1 - (-\frac{1}{2})) = -1 =$ the hypercharge of a left-handed electron by the relation of Gell-Mann Nishijima,

$g_W \equiv$ a unit-free real number > 0 , measuring the probability of an interaction between $(\tau \cdot \mathbf{b}_\mu)$ and the left-handed electron e_L^- along with a neutrino ν that are contained in L in $L_{leptons}$,

$$\begin{aligned} \tau \cdot \mathbf{b}_\mu &\equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} b_\mu^{(1)} + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} b_\mu^{(2)} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} b_\mu^{(3)}, \\ V(z^* z) &: = -a(z^* z) + b(z^* z)^2, \quad a, b > 0, \text{ in units:} \\ \frac{J^2}{m^6} &= (Js)^2 \cdot \left(\frac{1/s^2}{m^6} \right) + (Js)^2 (m^3 s)^2 \cdot \left(\frac{1/s^4}{m^{12}} \right), \end{aligned} \quad (2.2)$$

implying that

$$z(t, x, y, z) = \omega(t, x, y, z) / m^3$$

is the angular frequency density of the (electromagnetic, by the combined 4-manifold, see [3]) wave associated with a Higgs particle.

$$\text{At } z = \begin{pmatrix} 0 \\ \sqrt{\frac{a}{b}} \end{pmatrix}, \text{ one has } V = 0,$$

which is frame invariant since any relativistic contraction/expansion would not alter 0.

$$\begin{aligned} \text{Set } \omega_0 &: = \sqrt{\frac{a}{b}} m^3; \\ &\text{then } \frac{\omega_0}{\sqrt{2}} \text{ minimizes } V. \\ \text{Define } \frac{\eta}{\sqrt{2}} &: = \omega - \frac{\omega_0}{\sqrt{2}}; \end{aligned}$$

then

$$\begin{aligned}
 m^6 V &= -a\omega^2 + b\omega^4 \text{ (by Equation (2.2))} \\
 &\equiv -a \left(\frac{\eta}{\sqrt{2}} + \frac{\omega_0}{\sqrt{2}} \right)^2 + b \left(\frac{\eta}{\sqrt{2}} + \frac{\omega_0}{\sqrt{2}} \right)^4 \\
 &= \left(\frac{\eta}{\sqrt{2}} \right)^2 \left(-a + 6b \cdot \frac{a}{2b} \right) + \text{the other terms } (***) \\
 &= a\eta^2 + (***) ,
 \end{aligned}$$

so that a substitution of $\left(\frac{\eta}{\sqrt{2}} \right)$ into Equation (2.1) yields

$$\begin{aligned}
 &\left(i\hbar \frac{\partial}{\partial x^\mu} \frac{\eta}{\sqrt{2}} \right)^* \left(i\hbar \frac{\partial}{\partial x_\mu} \frac{\eta}{\sqrt{2}} \right) - V \left(\frac{\eta}{\sqrt{2}} \right) \\
 &= \frac{1}{2} \left[\left(\hbar \frac{\partial}{\partial x^\mu} \eta \right)^* \left(\hbar \frac{\partial}{\partial x_\mu} \eta \right) - 2a\eta^2 \right] - (***) \\
 &= \frac{1}{2} \left[(E_{Higgs} - p_{Higgs}c)^2 - (m_{0,Higgs}c^2)^2 \right] - (***) .
 \end{aligned}$$

Then,

$$(\min L_{scalar}) \implies \left(m_{0,Higgs}c^2 = \sqrt{2a} \cdot \frac{\omega_0}{\sqrt{2}} = \hbar \cdot \frac{\omega_0}{\sqrt{2}} \right) . \quad (2.3)$$

Since the objective now is to assign the Higgs particle's rest energy $\hbar \cdot \frac{\omega_0}{\sqrt{2}}$ to the leptons, the time coordinate $t = \mu$ will suffice for the purpose, i.e., to consider only A_t^M , $b_t^{(1)}$, $b_t^{(2)}$, and $b_t^{(3)}$. Substituting $b_t^{(1)}$, $b_t^{(2)}$, and $\phi_0 = \frac{\omega_0}{\sqrt{2}}$ into Equation (2.1), we have

$$\begin{aligned}
 &\left(\frac{ig_W \hbar}{2} \tau \cdot \mathbf{b}_t \phi_0 \right)^* \left(\frac{ig_W \hbar}{2} \tau \cdot \mathbf{b}_t \phi_0 \right) \quad (2.4) \\
 &= \frac{1}{4} g_W^2 \hbar^2 \phi_0^2 \begin{pmatrix} 0 & b_t^{(1)} + b_t^{(2)}i \\ b_t^{(1)} - b_t^{(2)}i & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & b_t^{(1)} - b_t^{(2)}i \\ b_t^{(1)} + b_t^{(2)}i & 0 \end{pmatrix} \\
 &= \frac{1}{2} g_W^2 \hbar^2 \phi_0^2 \left(\left| \frac{b_t^{(1)} + b_t^{(2)}i}{\sqrt{2}} \right|^2 + \left| \frac{b_t^{(1)} - b_t^{(2)}i}{\sqrt{2}} \right|^2 \right) \\
 &\equiv \frac{1}{2} g_W^2 \hbar^2 \phi_0^2 (|W_t^-|^2 + |W_t^+|^2) = g_W^2 \hbar^2 \phi_0^2 |W_t^\pm|^2 ,
 \end{aligned}$$

implying that

$$m_{0,W^\pm}c^2 = g_W \hbar \cdot \frac{\omega_0}{\sqrt{2}} . \quad (2.5)$$

Substituting W^\pm into the Lagrangian of the left-handed leptons (Equation (1.1)), we have

$$\begin{aligned}
& i\gamma^\mu \cdot \frac{ig_W}{2} \bar{L} \begin{pmatrix} 0 & b_\mu^{(1)} - b_\mu^{(2)}i \\ b_\mu^{(1)} + b_\mu^{(2)}i & 0 \end{pmatrix} L \\
&= -\frac{\sqrt{2}}{2} g_W \gamma^\mu \bar{L} \begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix} L \\
&= -\frac{\sqrt{2}}{2} g_W \gamma^\mu \begin{pmatrix} \bar{\nu}_L & \bar{e}_L \end{pmatrix} \begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \\
&= -\frac{\sqrt{2}}{2} g_W \gamma^\mu (\bar{\nu}_L W_\mu^+ e_L + \bar{e}_L W_\mu^- \nu_L) \\
&= -\frac{\sqrt{2}}{4} g_W \left(\bar{\nu} \gamma^\mu \begin{pmatrix} O_2 & O_2 \\ O_2 & 2I_2 \end{pmatrix} e W_\mu^+ + \bar{e} \gamma^\mu \begin{pmatrix} O_2 & O_2 \\ O_2 & 2I_2 \end{pmatrix} \nu W_\mu^- \right) \\
&= -\sqrt{\frac{G_F \cdot (m_{0,W^\pm c^2})^2}{\sqrt{2}}} \\
&\quad \cdot \left(\bar{\nu} \gamma^\mu \begin{pmatrix} O_2 & O_2 \\ O_2 & 2I_2 \end{pmatrix} e W_\mu^+ + \bar{e} \gamma^\mu \begin{pmatrix} O_2 & O_2 \\ O_2 & 2I_2 \end{pmatrix} \nu W_\mu^- \right) \\
&\quad \text{(by low-energy correspondence, see [4], p. 110;} \\
G_F &\equiv \text{Fermi constant} \approx 1.17 \times 10^{-5} (GeV)^{-2}.
\end{aligned}$$

Thus, $\frac{g_W^2}{8} = \frac{G_F \cdot (m_{0,W^\pm c^2})^2}{\sqrt{2}}$, but $m_{0,W^\pm c^2} = g_W \hbar \cdot \frac{\omega_0}{\sqrt{2}}$ (by Equation (2.5)), so

$$\begin{aligned}
\hbar \cdot \frac{\omega_0}{\sqrt{2}} &= \left[\frac{\sqrt{2}}{8} \times 1.17^{-1} \times 10^5 (GeV)^2 \right]^{1/2} \\
&= [0.177 \times 0.85 \times 10^5]^{1/2} GeV \\
&= \sqrt{0.151 \times 10^5} GeV \\
&= \sqrt{15109} GeV \approx 122.92 GeV = m_{0,Higgs} c^2 \text{ by Equation (2.3).}
\end{aligned}$$

2.2. The Weinberg Angle

We next transform A_μ^M and $b_\mu^{(3)}$ into $A_{0,\mu}$ and $Z_{0,\mu}$ by setting $\mu = t$,

$$\begin{pmatrix} Z_{0,t} \\ A_{0,t} \end{pmatrix} : = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{g_E^2 + g_W^2}} \begin{pmatrix} g_W & -g_E \\ g_E & g_W \end{pmatrix} \begin{pmatrix} b_t^{(3)} \\ A_t^M \end{pmatrix}, \text{ or} \\ \begin{pmatrix} b_t^{(3)} \\ A_t^M \end{pmatrix} = \frac{\sqrt{2}}{\sqrt{g_E^2 + g_W^2}} \begin{pmatrix} g_W & g_E \\ -g_E & g_W \end{pmatrix} \begin{pmatrix} Z_{0,t} \\ A_{0,t} \end{pmatrix}, \quad (2.6)$$

where the factor of $\sqrt{2}$ is needed (overlooked in [4]) in anticipation of the vanishing of \mathbf{A}^M . Then L_{scalar} has

$$\left\{ \left[\frac{ig_E \hbar}{2} A_t^M \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{ig_W \hbar}{2} b_t^{(3)} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right] \begin{pmatrix} 0 \\ \phi_0 \end{pmatrix} \right\}^* \quad (2.7)$$

$$\cdot \left\{ \left[\frac{ig_E \hbar}{2} A_t^M \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{ig_W \hbar}{2} b_t^{(3)} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right] \begin{pmatrix} 0 \\ \phi_0 \end{pmatrix} \right\} \quad (2.8)$$

$$= \frac{1}{4} \left(\hbar \cdot \frac{\omega_0}{\sqrt{2}} \right)^2 \left(g_E A_t^M - g_W b_t^{(3)} \right)^2; \quad (2.9)$$

substituting Equation (2.6) into the above expression (2.9) and taking the square-root, one then has

$$\frac{1}{2} \left(\hbar \cdot \frac{\omega_0}{\sqrt{2}} \right) \cdot \frac{\sqrt{2}}{\sqrt{g_E^2 + g_W^2}} [g_E (-g_E Z_{0,t} + g_W A_{0,t}) \quad (2.10)$$

$$-g_W (g_W Z_{0,t} + g_E A_{0,t})] \quad (2.11)$$

$$= -\frac{\sqrt{2}}{2} \left(\hbar \cdot \frac{\omega_0}{\sqrt{2}} \right) \sqrt{g_E^2 + g_W^2} Z_{0,t}, \quad (2.12)$$

i.e.,

$$m_{0,Z_0} c^2 = \frac{\sqrt{2}}{2} \sqrt{g_E^2 + g_W^2} \hbar \cdot \frac{\omega_0}{\sqrt{2}},$$

$$\text{and } m_{0,A_0} c^2 = 0.$$

However, the above matrix multiplication of

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ \phi_0 \end{pmatrix} \text{ is invalid,}$$

since (a) $\begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix}$ is not a vector in \mathbb{R}^3 , as noted earlier, and (b) the identification of $z \equiv \phi_+ + \phi_0 i$ as a "complex (iso)doublet" would be in disagreement with the meaning of the Pauli matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, which does not correspond to $1-i$. I.e., the cancellation of $g_E g_W A_{0,t} - g_W g_E A_{0,t}$ in the expression (2.10) could not have been achieved had $\begin{pmatrix} 0 \\ \phi_0 \end{pmatrix}$ been treated as a scalar as in Equation (2.4), for then we would have had

$$\begin{aligned} & \left\{ \left[\frac{ig_E \hbar}{2} A_t^M \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{ig_W \hbar}{2} b_t^{(3)} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right] \phi_0 \right\}^* \\ & \cdot \left\{ \left[\frac{ig_E \hbar}{2} A_t^M \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{ig_W \hbar}{2} b_t^{(3)} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right] \phi_0 \right\} \\ & = \frac{1}{4} \left(\hbar \cdot \frac{\omega_0}{\sqrt{2}} \right)^2 \left[\left(g_E A_t^M + g_W b_t^{(3)} \right)^2 + \left(g_E A_t^M - g_W b_t^{(3)} \right)^2 \right], \\ & \text{with an extra term of } \left(g_E A_t^M + g_W b_t^{(3)} \right)^2. \end{aligned}$$

As such, to circumvent the problem caused by the sign of the a_{11} entry of $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, the literature had no choice but setting up a form like $\begin{pmatrix} 0 \\ \phi_0 \end{pmatrix}$, yet this is in vain: Recall that Pauli spin matrices enter into the calculation of the probabilities of the two spin states of an electron, up or down; to cast these two linearly dependent vectors as separated by π radians into two linearly independent vectors as separated by $\pi/2$ radians, one is led to make a frame transformation of the first three columns of σ_x , σ_y , and σ_z into the second three columns of the three matrices via two spatial $\pi/2$ rotations (see [3]). That is, Pauli matrices describe momenta of an electromagnetic field energy flows at three particular points around two semi-circles osculating at a right angle, where $\begin{pmatrix} 0 \\ i \end{pmatrix} \equiv \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ refers to a momentum direction pointing to the z -axis.

As such, $z \equiv \phi_+ + \phi_0 i \equiv \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix}$, devoid of any directional intent, cannot be operated by the Pauli matrix $\sigma_z := \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. To be sure, the artificial setup of $\begin{pmatrix} 0 \\ \phi_0 \end{pmatrix}$ is equivalent to altering σ_z to $\begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$, apparently illegitimate. In short, the linear transformation (2.6) fails to remove A_0 as a recipient of a

rest mass from the very same Higgs particle that imparts rest masses to W^\pm and Z_0 . As such, photon, of no rest mass, is not the claimed gauge boson A_0 of electromagnetism, and the electroweak integration breaks down, disassembling the Standard Model.

As a parenthetical passing note, we have previously generalized Pauli matrix $\sigma_y := \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ to $\begin{pmatrix} 0 & a - bi \\ a + bi & 0 \end{pmatrix}$ with $a^2 + b^2 = 1$, which by virtue of squaring to I_2 still satisfies the mass-shell equation. We factored the mass-shell equation into $(RE + iKE)(RE - iKE) = RE^2 + KE^2 = E^2$, implying that anti-particles take conjugate forms, and that turns out precisely to be the form of W^\pm , in discord with Dirac's rendition of $E = \pm\sqrt{RE^2 + KE^2}$, which by the way was also invalid since the quaternion property of Pauli matrices only applies to one scalar function, not the 4-spinors, where a $(\frac{\partial}{\partial t})$ as the operator of a specific electron wavefunction would operate on two different wavefunctions (see [3]).

3. Conclusion

In this paper, we derived the Higgs mass to be 123 GeV, in close proximity to the well-known LHC's finding of 125 GeV; at the same time, we showed the photon in the Standard Electroweak Model to possess a rest mass, in support of the reservation of the correctness of $U(1) \times SU(2)$ as remarked in [4]. Correct physical modeling facilitates technological advancement, which hopefully can be achieved through the research community's drive for *New Physics*.

References

- [1] S. Bhattacharjee and P. Majumdar, Gauge-free electroweak theory: Radiative effects, *Phys. Rev. D* **83** (2011), # 085019 (1-7).
- [2] R.M. Capdevilla, A. Delgado, and A. Martin, Light stops in a minimal $U(1)_X$ extension of the MSSM, *Phys. Rev. D* **92** (2015), # 115020 (1-8).
- [3] G.L. Light, Remarks on the standard model, *Int. J. Appl. Math.* **28**, No 6 (2015), 651-666; doi: 10.12732/ijam.v28i6.2 .
- [4] C. Quigg, *Gauge Theories of the Strong, Weak, and Electromagnetic Interactions*, Princeton University Press, Princeton (2013).

