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STUDY OF A ONE-DIMENSIONAL UNSTEADY GAS DYNAMIC PROBLEM BY ADOMIAN DECOMPOSITION METHOD

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Abstract: The system of gas dynamic equations governing the motion of one-dimensional unsteady adiabatic flow of a perfect gas in planer, cylindrical and spherical symmetry is solved successfully by applying the Adomian decomposition method under the exponential initial conditions. The solution of the system of equation is computed up to the five components of the decomposition series. The variation of the approximate velocity, density and pressure of the fluid motion with position and time is studied. It is found that there exists discontinuity or shock wave in the distribution of flow variables. The solution of system of gas dynamic equations by Adomian decomposition method is convergent for a domain of position and time. The decomposition method provides the variation of flow-variables with position and time separately which was not possible in similarity method.

AMS Subject Classification: 35L65, 76N15, 76M99
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1. Introduction

The conservation laws for the motion of a gas in its mathematical formulation is written as a system of gas dynamic equations for mass, momentum and

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energy. This system of gas dynamic equation is complemented by the equation of the state of the gas. The solution of the system of one-dimensional unsteady gas dynamic problem under the suitable initial condition is an important field of study. The solution of the system of gas dynamic equations are studied in literature and is solved by the different methods. One of the important and widely used method to solve the equations is similarity method but it needs the much computational work, knowledge of geometrical aspects of the problem and give solution in term of similarity variable, not in term of space and time variables directly, [1]. Recently, considerable amount of research work has been invested in applying the Adomian decomposition method to a wide class of linear and non-linear ordinary and partial differential equation as well as integral equation [2, 3, 4, 5, 6, 7, 8, 9, 10]. The Adomian decomposition method was introduced and developed by George Adomian [11] − [12] and is well addressed in the literature. This method has been used in the solution of scalar gas dynamic equation [13] − [14]. The system of gas dynamic equation has been solved by the Adomian decomposition method in two dimension for polytropic gas [15]. But the system of gas dynamic equations for the unsteady adiabatic flow of a perfect gas in one dimension for all the three type of symmetry namely planer, cylindrical and spherical has not been studied.

In the present work, we have solved the system of non-linear one-dimensional unsteady gas dynamic equations for adiabatic flow of a perfect gas in all the three symmetry under the variable initial condition by applying the Adomian decomposition method [12], [16] and [17]. The convergence and accuracy of the solution is discussed. The variation of flow-variables with position and time is presented which show that there exist a discontinuity or shock wave in the flow field. All the computations are performed using *Mathematica* 9.

2. Basic equations and initial conditions

The system of gas dynamic equations governing the motion of a one-dimensional unsteady adiabatic flow of a perfect gas in planer, cylindrical and spherical symmetry is given by [1]

$$\frac{\partial u^{'}}{\partial t^{'}} + u^{'}\frac{\partial u^{'}}{\partial r^{'}} + \frac{1}{\rho^{'}}\frac{\partial p^{'}}{\partial r^{'}} = 0, \tag{1}$$

$$\frac{\partial \rho^{'}}{\partial t^{'}} + u^{'}\frac{\partial \rho^{'}}{\partial r^{'}} + \rho^{'}\frac{\partial u^{'}}{\partial r^{'}} + (\nu - 1)\frac{\rho^{'} u^{'}}{r^{'}} = 0, \tag{2}$$

$$\frac{\partial}{\partial t^{'}}(\frac{p^{'}}{\rho^{'\gamma}}) + u^{'}\frac{\partial}{\partial r^{'}}(\frac{p^{'}}{\rho^{'\gamma}}) = 0, \tag{3}$$

where $u^{'}$, $\rho^{'}$ and $p^{'}$ are the velocity density and pressure respectively at position $r^{'}$ and time $t^{'}$, γ is adiabatic exponent, ν is symmetry parameter which takes the values 1, 2 and 3 for the planer, cylindrical and spherical symmetry. The initial conditions for the motion is taken as

$$u^{'}(r^{'},0) = u_0 e^{\frac{-r^{'}}{l_0}},\ \rho^{'}(r^{'},0) = \rho_0 e^{\frac{-r^{'}}{l_0}},\ p^{'}(r^{'},0) = -\frac{\rho_0 u_0^2}{3} e^{\frac{-3r^{'}}{l_0}}, \tag{4}$$

where $0 \leq r^{'} < \infty$, $t^{'} \geq 0$ and l_0, u_0 and ρ_0 are dimensional constant. The initial condition is consistent with the equation of motion (1). The equation of state for the perfect gas is $p^{'} = \rho^{'} RT$ where R and T are gas constant and temperature respectively. The technique to make the above equations dimensionless consists of introducing a simple change of variables $t^{'} = t_0 t$, $r^{'} = l_0 r$, $u^{'} = u_0 u$, $\rho^{'} = \rho_0 \rho$ and $p^{'} = \rho_0 u_0^2 p$, where $t^{'}$, $r^{'}$, $u^{'}$, $\rho^{'}$ and $p^{'}$ are the original dimensional variables, t, r, u, ρ and p are corresponding dimensionless variables, and t_0, l_0, u_0 and ρ_0 are characteristic time, characteristic length, characteristic velocity and characteristic density. Under the transformation the equations of motion of one-dimensional unsteady adiabatic flow of a perfect gas in planer, cylindrical and spherical symmetry reduces as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0, \tag{5}$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + (\nu - 1)\frac{\rho u}{r} = 0, \tag{6}$$

$$\frac{\partial}{\partial t}(\frac{p}{\rho^{\gamma}}) + u\frac{\partial}{\partial r}(\frac{p}{\rho^{\gamma}}) = 0, \tag{7}$$

and the initial conditions in non-dimensional form become

$$u(r,0) = e^{-r},\ \ \rho(r,0) = e^{-r},\ \ p(r,0) = -(1/3)e^{-3r}, \tag{8}$$

taking the Strouhal number $S = l_0/u_0 t_0 = 1$ to consider the transient process.

3. Basic idea of Adomian decomposition method

In Adomian decomposition method, we consider the partial differential equation in an operator form

$$Lu + Ru + Nu = g, \tag{9}$$

where L is highest order linear operator which is assumed to be invertible and R is linear operator of order less than order of L, N is non-linear operator and g is a source term. We apply the inverse operator L^{-1} to both sides of equation (9) and using the given condition, we obtained

$$u = f - L^{-1}(Ru) - L^{-1}(Nu), \tag{10}$$

where the function f represents the terms arising from integrating the source term g and using the given condition, all are assumed to be prescribed. The standard Adomian decomposition method suggests the solution u in form of infinite series of the components, given by

$$u = \sum_{n=0}^{\infty} u_n, \tag{11}$$

where u_n, $n \geq 0$ are components of u. The non-linear operator N is decomposed in series form

$$N = \sum_{n=0}^{\infty} A_n, \tag{12}$$

where A_n are called Adomian polynomials. The aim of decomposition method is to determine the components $u_0, u_1, u_2 ...$; recursively and elegantly. The Adomian polynomials can be generated for all form of non-linearity [18] – [19]. The Adomian polynomials A_n are generated according to the algorithms previously published in [19] and standard Adomian decomposition scheme introduced in [12] – [20]. On substituting Eq. (11) into Eq. (10) we get

$$\sum_{n=0}^{\infty} u_n = f - L^{-1}(R \sum_{n=0}^{\infty} u_n) - L^{-1}(N \sum_{n=0}^{\infty} u_n). \tag{13}$$

Following the Adomian analysis in Eq. (13), the non-linear partial differential Eq. (9) with the condition is transformed into a set of recursive relations

$$u_0 = f, \tag{14}$$

$$u_{k+1} = -L^{-1}(Ru_k) - L^{-1}(Nu_k); \quad k \geq 0. \tag{15}$$

From Eqs. (14) and (15), by putting the components u_n, $n \geq 0$ in Eq. (11) the solution u is obtained readily.

4. Solution of the system of gas dynamic equations

Firstly we eliminate $\partial\rho/\partial t$ in Eq. (7) by using Eq. (6) then we write the basic Eqs. (5), (6) and (7) in the following operator form

$$L_t u + uL_r u + (1/\rho)L_r p = 0, \tag{16}$$

$$L_t \rho + uL_r \rho + \rho L_r u + (\nu - 1)\rho u/r = 0, \tag{17}$$

$$L_t p + \gamma p L_r u + uL_r p + \gamma(\nu - 1)pu/r = 0, \tag{18}$$

where L_t and L_r are the partial differential operators given by $L_t = \partial/\partial t$ and $L_r = \partial/\partial r$. Then following the standard Adomian method, Eqs. (16)–(18) can be written as

$$u(r,t) = e^{-r} - L_t^{-1}(uu_r) - L_t^{-1}(p_r/\rho), \tag{19}$$

$$\rho(r,t) = e^{-r} - L_t^{-1}(u\rho_r) - L_t^{-1}(\rho u_r) - (\nu - 1)L_t^{-1}(\rho u/r), \tag{20}$$

$$p(r,t) = \frac{-e^{-3r}}{3} - \gamma L_t^{-1}(pu_r) - L_t^{-1}(up_r) - \gamma(\nu - 1)L_t^{-1}(pu/r). \tag{21}$$

Let us assume the solution u, ρ and p of the Eqs. (16)–(18) in the form of decomposition series as

$$u(r,t) = \sum_{n=0}^{\infty} u_n(r,t), \tag{22}$$

$$\rho(r,t) = \sum_{n=0}^{\infty} \rho_n(r,t), \tag{23}$$

$$p(r,t) = \sum_{n=0}^{\infty} p_n(r,t). \tag{24}$$

The non-linear term uu_r, p_r/ρ, $u\rho_r$, ρu_r, pu_r and up_r in Eqs. (19)–(21) is expressed in series form as

$$u\rho_r = \sum_{n=0}^{\infty} A_n, \quad \rho u_r = \sum_{n=0}^{\infty} B_n, \tag{25}$$

$$u u_r = \sum_{n=0}^{\infty} C_n, \quad \frac{p_r}{\rho} = \sum_{n=0}^{\infty} D_n, \tag{26}$$

$$p u_r = \sum_{n=0}^{\infty} E_n, \quad u p_r = \sum_{n=0}^{\infty} F_n, \tag{27}$$

where A_n, B_n, C_n, D_n, E_n and F_n are Adomian polynomials. Substituting Eqs. (22)–(24) and (25)–(27) into both sides of Eqs. (19)–(21) we find

$$\sum_{n=0}^{\infty} u_n(r,t) = e^{-r} - L_t^{-1}(\sum_{n=0}^{\infty} C_n) - L_t^{-1}(\sum_{n=0}^{\infty} D_n), \tag{28}$$

$$\begin{aligned}\sum_{n=0}^{\infty} \rho_n(r,t) = e^{-r} - L_t^{-1}(\sum_{n=0}^{\infty} A_n) - L_t^{-1}(\sum_{n=0}^{\infty} B_n)\\ -(\nu-1)L_t^{-1}(\sum_{n=0}^{\infty} \rho_n \sum_{n=0}^{\infty} u_n)/r,\end{aligned} \tag{29}$$

$$\begin{aligned}\sum_{n=0}^{\infty} p_n(r,t) = -e^{-3r}/3 - \gamma L_t^{-1}(\sum_{n=0}^{\infty} E_n) - L_t^{-1}(\sum_{n=0}^{\infty} F_n)\\ -\gamma(\nu-1)L_t^{-1}(\sum_{n=0}^{\infty} p_n \sum_{n=0}^{\infty} u_n)/r.\end{aligned} \tag{30}$$

Following the decomposition method in Eqs. (28)–(30) we obtained the recursive relation for gas dynamic Eqs. (16)–(18) in an operator form as

$$u_0(r,t) = e^{-r}, \tag{31}$$

$$u_1(r,t) = -L_t^{-1}(C_0) - L_t^{-1}(D_0), \tag{32}$$

$$u_{k+1}(r,t) = -L_t^{-1}(C_k) - L_t^{-1}(D_k), \tag{33}$$

$$\rho_0(r,t) = e^{-r}, \tag{34}$$

$$\rho_1(r,t) = -L_t^{-1}(A_0) - L_t^{-1}(B_0) - (\nu-1)L_t^{-1}(p_0 u_0/r), \tag{35}$$

$$\rho_{k+1}(r,t) = -L_t^{-1}(A_k) - L_t^{-1}(B_k) - (\nu-1)L_t^{-1}(\rho_k u_k/r), \tag{36}$$

$$p_0(r,t) = -e^{-3r}/3, \tag{37}$$

$$p_1(r,t) = -\gamma L_t^{-1}(E_0) - L_t^{-1}(F_0) - \gamma(\nu-1)L_t^{-1}(p_0 u_0/r), \tag{38}$$

$$p_{k+1}(r,t) = -\gamma L_t^{-1}(E_k) - L_t^{-1}(F_k) - \gamma(\nu-1)L_t^{-1}(p_k u_k/r), \tag{39}$$

where $k \geq 1$.

We can obtain the solution of gas dynamic equations from recursive Eqs. (31)–(39) in series form provided the Adomian polynomials are known. The Adomian polynomials are calculated as follows:

$$\sum_{n=0}^{\infty} A_n = u\rho_r = (u_0 + u_1 + u_2 + u_3 + \cdots +) \times (\rho_{0r} + \rho_{1r} + \rho_{2r} + \rho_{3r} + \cdots +),$$

we now, compare the term with subscript of same order from both side of above equation, then Adomian polynomial A_n is given by

$$A_n = \sum_{k=0}^{n} u_k \rho_{(n-k)r}, \quad n = 0, 1, 2....$$

Similarly, we can obtain Adomian polynomial B_n, C_n, F_n, as

$$B_n = \sum_{k=0}^{n} \rho_k u_{(n-k)r}, \quad n = 0, 1, 2....$$

$$C_n = \sum_{k=0}^{n} u_k u_{(n-k)r}, \quad n = 0, 1, 2....$$

$$F_n = \sum_{k=0}^{n} u_k P_{(n-k)r}, \quad n = 0, 1, 2....$$

and,

$$\sum_{n=0}^{\infty} D_n = \frac{p_r}{\rho} = \left(\frac{p_{0r} + p_{1r} + p_{2r} + p_{3r} + \cdots}{\rho_0 + \rho_1 + \rho_2 + \rho_3 + \cdots}\right),$$

so, comparing the term with subscript of same order from both sides of above equation, we get

$$\begin{aligned}
D_0 &= \frac{p_{0r}}{\rho_0}, \\
D_1 &= -\frac{p_{0r}\rho_1}{\rho_0^2} + \frac{p_{1r}}{\rho_0}, \\
D_2 &= -\frac{p_{0r}\rho_2}{\rho_0^2} + \frac{p_{0r}\rho_1^2}{\rho_0^3} - \frac{p_{1r}\rho_1}{\rho_0^2} + \frac{p_{2r}}{\rho_0}, \\
D_3 &= -\frac{p_{0r}\rho_3}{\rho_0^3} - \frac{p_{0r}\rho_1^3}{\rho_0^4} + \frac{2p_{0r}\rho_1\rho_2}{\rho_0^3} + \frac{p_{3r}}{\rho_0} - \frac{p_{0r}\rho_1}{\rho_0^2} - \frac{p_{1r}\rho_2}{\rho_0^2},
\end{aligned}$$

and so on. Similarly we can write E_n also. Using the Adomian polynomials A_n, B_n, C_n, D_n, E_n and F_n into the recursive relations (32)–(33), (35)–(36) and (38)–(39), we can compute the components of the velocity, the density and the pressure and then the solution $u(r,t)$, $\rho(r,t)$ and $p(r,t)$ of gas dynamic equations are given by the Eqs. (22)–(24).

5. Result and Discussion

The distribution of flow variables in the flow-field is calculated by the Eqs. (22)–(24) for $\gamma = 1.4$ and $\nu = 1, 2, 3$ for planer, cylindrical and spherical symmetry. For each case of symmetry, the five components of the velocity, density and pressure are computed from the recursive Eqs. (31)–(39) and approximate velocity, density and pressure are given in form of tables and figures.

5.1. The velocity components for planer symmetry $\nu = 1$ are computed as follows:

$$\begin{aligned}
u_0 &= e^{-r},\\
u_1 &= 0,\\
u_2 &= (-2.93333e^{-3r} + e^{-4r})t^2,\\
u_3 &= (-6.93332e^{-4r} + 1.66667e^{-5r})t^3,\\
u_4 &= \{(-8.14682 + 0.78222r)e^{-5r} + (2.96111 - 0.31111r)e^{-6r}\}t^4,
\end{aligned}$$

and so on. Therefore, the velocity u is given by putting the value of u_0, u_1, u_2, u_3, $u_4 \cdots$, in to the Eq. (22). For numerical computation we truncate the series after five term and the approximate velocity $\Phi_{5,u}$ is given by

$$\Phi_{5,u} = \sum_{n=0}^{4} u_n = e^{-r} + 0 + (-2.93333e^{-3r} + e^{-4r})t^2$$
$$+(-6.93332e^{-4r} + 1.66667e^{-5r})t^3 + \{(-8.14682 + 0.78222r)$$
$$+(2.96111 - 0.31111r)e^{-6r}\}t^4.$$

The approximate velocity $\Phi_{5,u}$ for $\nu = 1$ is presented in Table 1 and Fig. 1a. Similarly, for $\nu = 2$ and $\nu = 3$ we have computed the five velocity components for different value of r and t and approximate velocity $\Phi_{5,u}$ is shown in Tables 4, 7 and Figs. 2a, 3a respectively.

5.2. The density components for planer symmetry $\nu = 1$ are calculated as follows

$$\begin{aligned}
\rho_0 &= e^{-r},\\
\rho_1 &= 2e^{-2r}t,\\
\rho_2 &= 2e^{-3r}t^2,\\
\rho_3 &= (3.95555e^{-4r} + 1.66666e^{-5r})t^3,\\
\rho_4 &= (-15.88887e^{-5r} + 8.00000e^{-6r})t^4,
\end{aligned}$$

and so on. Therefore, the density distribution ρ in the flow-field is given by the Eq. (23). The approximate density $\Psi_{5,\rho}$ after truncation up to five term of Eq. (23) is given by

$$\Psi_{5,\rho} = \sum_{n=0}^{4} \rho_n = e^{-r} + 2e^{-2r}t + 2e^{-3r}t^2 + (3.95555e^{-4r}$$
$$+1.66666e^{-5r})t^3 + (-15.88887e^{-5r} + 8.00000e^{-6r})t^4.$$

This approximate density $\Psi_{5,\rho}$ for $\nu = 1$ is presented in Table 2 and Fig. 1b. Similarly, we have computed the approximate density $\Psi_{5,\rho}$ for $\nu = 2$ and $\nu = 3$ and given into Tables 5, 8 and Figs. 2b and 3b respectively for different values of r and t.

5.3. The pressure components for planer symmetry $\nu = 1$ are computed as follows

$$\begin{aligned}
p_0 &= -e^{-3r}/3,\\
p_1 &= (-1.46667e^{-4r})t,\\
p_2 &= (-3.96000e^{-5r})t^2,\\
p_3 &= \{(-8.44801 + .52147r)e^{-6r} + (1.22227r)e^{-7r}\}t^3,\\
p_4 &= \{(-1.97074 + 0.96474r)e^{-7r} - (4.86444 + 0.37333r)e^{-8r}\}t^4,
\end{aligned}$$

and so on. So, the pressure distribution p in the flow-field is obtained by Eq. (24). The approximate pressure $\chi_{5,p}$ after truncation up to five term of the decomposition series (24) is

$$\begin{aligned}
\chi_{5,p} = \sum_{n=0}^{4} p_n = (-0.3333e^{-3r}) + (-1.4667e^{-4r})t + (-3.9600e^{-5r})t^2\\
+\{(-8.4480 + .5215r)e^{-6r} + (1.2223r)e^{-7r}\}t^3\\
+\{(-1.9707 + 0.9647)e^{-7r} - (4.8644 + 0.3733r)e^{-8r}\}t^4.
\end{aligned}$$

The numerical value of approximate pressure $\chi_{5,p}$ for $\nu = 1$ is given in Table 3 and plotted in Fig. 1c. Similarly, the approximate pressure $\chi_{5,p}$ for $\nu = 2$ and $\nu = 3$ is given in Tables 6, 9 and Figs. 2c and 3c respectively for different values of r and t.

t	r	u_0	u_1	u_2	u_3	u_4	$\Phi_{5,u}$
0.5	0.5	6.0653e-01	0	-1.2980e-01	-1.0019e-01	-3.1059e-02	3.4549e-01
	1	3.6789e-01	0	-3.1932e-02	-1.4470e-02	-2.6985e-03	3.1879e-01
	2	1.3534e-01	0	-1.7339e-03	-2.8128e-04	-1.7779e-05	1.3330e-01
1	0.5	6.0653e-01	0	-5.1918e-01	-8.0152e-01	-4.9695e-01	-1.2111e-00
	1	3.6789e-01	0	-1.2773e-01	-1.1576e-01	-4.3054e-02	8.1341e-01
	2	1.3534e-01	0	-6.9355e-03	-2.2502e-03	-2.8447e-04	1.2587e-01
1.5	0.5	6.0653e-01	0	-1.1684e-00	-2.7051e-00	-2.5158e-00	-5.7825e-00
	1	3.6789e-01	0	-2.8738e-01	-3.9069e-01	-2.1796e-01	-5.2815e-01
	2	1.3534e-01	0	-1.5605e-02	-7.5945e-03	-1.4401e-03	1.1067e-01
2	0.5	6.0653e-01	0	-2.0767e-00	-6.4121e-00	-7.9511e-00	-1.5834e01
	1	3.6789e-01	0	-5.1091e-01	-9.2607e-01	-6.8886e-01	-7.7580e-00
	2	1.3534e-01	0	-5.1091e-01	-1.8002e-02	-4.5518e-03	8.5040e-02

Table 1: Variation of the velocity components and approximate velocity $\Phi_{5,u}$ with different values of r and t for planer symmetry $\nu = 1$ and $\gamma = 1.4$.

t	r	ρ_0	ρ_1	ρ_2	ρ_3	ρ_4	$\Psi_{5,\rho}$
0.5	0.5	6.0653e-01	3.6788e-01	1.6735e-01	1.8605e-02	-5.6621e-02	1.1037e-00
	1	3.6789e-01	1.3534e-01	3.7340e-02	1.6073e-03	-5.4518e-03	5.3671e-01
	2	1.3534e-01	1.8316e-02	1.8591e-03	1.3186e-05	-4.2013e-05	1.5548e-01
1	0.5	6.0653e-01	7.3576e-01	6.6939e-01	1.4884e-01	-9.0594e-01	1.2546e-00
	1	3.6789e-01	2.7067e-01	1.4936e-01	1.2858e-02	-8.7229e-02	7.1354e-01
	2	1.3534e-01	3.6631e-02	7.4363e-03	1.0546e-04	-6.7220e-04	1.7884e-01
1.5	0.5	6.0653e-01	1.1036e-00	1.5061e-00	5.0233e-01	-4.5863e-00	-8.6771e-01
	1	3.6789e-01	4.0601e-01	3.3606e-01	4.3396e-02	-4.4159e-01	7.1175e-01
	2	1.3534e-01	5.4947e-02	1.6732e-02	3.5601e-04	-3.4030e-03	2.0397e-01
2	0.5	6.0653e-01	1.4715e-00	2.6776e-00	1.1907e-00	-1.4495e+1	-8.5488e-00
	1	3.6789e-01	5.4134e-01	5.9745e-01	1.0286e-01	-1.3957e-00	2.1387e-01
	2	1.3534e-01	7.3263e-02	2.9745e-02	8.4388e-04	-1.0755e-02	2.2843e-01

Table 2: Variation of the density components and approximate density $\Psi_{5,\rho}$ with different value of r and t for planer symmetry $\nu = 1$ and $\gamma = 1.4$.

t	r	p_0	p_1	p_2	p_3	p_4	$\chi_{5,p}$
0.5	0.5	-7.4377e-02	-9.9246e-02	-8.1264e-02	-5.1288e-02	-8.5912e-03	-3.1477e-01
	1	-1.6596e-02	-1.3432e-02	-6.6706e-03	-2.4763e-03	-1.6715e-04	-3.9341e-02
	2	-8.2625e-04	-2.4601e-04	-4.4946e-05	-5.7242e-06	-4.1609e-08	-1.1230e-03
1	0.5	-7.4377e-02	-1.9849e-01	-3.2506e-01	-4.1030e-01	-1.3746e-01	-1.1457e-01
	1	-1.6596e-02	-2.6863e-02	-2.6682e-02	-1.9810e-02	-2.6744e-03	-9.2625e-02
	2	-8.2625e-04	-4.9201e-04	-1.7978e-04	-4.5794e-05	-6.6574e-07	-1.5445e-03
1.5	0.5	-7.4377e-02	-2.9774e-01	-7.3136e-01	-1.3848e-00	-6.9589e-01	-3.1842e-00
	1	-1.6596e-02	-4.0294e-02	-6.0035e-02	-6.6859e-02	-1.3539e-02	-1.9732e-01
	2	-8.2625e-04	-7.3802e-04	-4.0451e-04	-1.5455e-04	-3.3703e-06	-2.1267e-03
2	0.5	-7.4377e-02	-3.9698e-01	-1.3002e-00	-3.2824e-00	-2.1993e-00	-7.2533e-00
	1	-1.6596e-02	-5.3726e-02	-1.0673e-01	-1.5848e-01	-4.2791e-02	-3.7832e-01
	2	-8.2625e-04	-9.8402e-04	-7.1914e-04	-3.6635e-04	-1.0652e-05	-2.9064e-03

Table 3: Variation of the pressure components and approximate pressure $\chi_{5,p}$ with different value of r and t for planer symmetry $\nu = 1$ and $\gamma = 1.4$.

Tables 1–3 show that contribution of higher order components in the velocity, density and pressure is comparatively small for all the time t and $r \geq 1$. The common ratio $\lambda_i = x_{i+1}/x_i < 1$, for $i = 0, 1, 2, 3$ and $x = u, \rho, p$; therefore, the series solution may converge by Adomian decomposition method for planer symmetry. Table 1 and Fig. 1a show that for planer symmetry, the velocity decreases with position for all the time t and $r > 0.5$ and increases steeply for $r < 0.5$ for all the time $t > 0.5$. This show that there exist a discontinuity surface or shock wave in the velocity distribution with respect to position. For the planar symmetry, Table 2 and Fig. 1b show that the density decreases with position for all time t except for $0 < r < 1$. For $0 < r < 1$ and $t > 1$, there exist a discontinuity, i.e. shock wave in distribution of density. Table 3 and Fig. 1c show that for the planer symmetry, the pressure increases steeply with position up to $r = 1$ and then it become almost constant for all the time t.

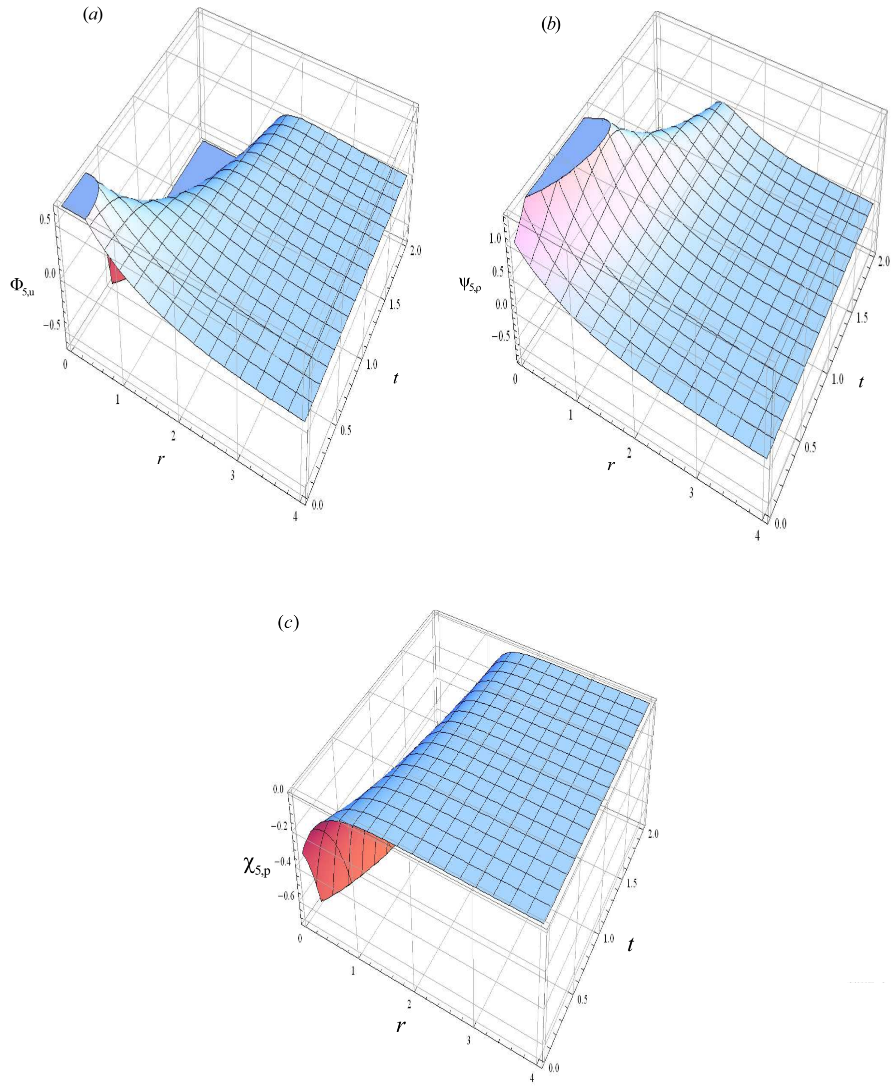

Figure 1: Distribution of flow variables: (**a**) approximate velocity, (**b**) approximate density, (**c**) approximate pressure, in flow field for planer symmetry

t	r	u_0	u_1	u_2	u_3	u_4	$\Phi_{5,u}$
0.5	0.5	6.0653e-01	0	-7.4377e-03	5.6166e-02	1.2359e-01	7.7885e-01
	1	3.6788e-01	0	-1.9700e-02	-9.1615e-03	-2.6825e-03	3.3634e-01
	2	1.3534e-01	0	-1.4295e-03	-2.4791e-04	-2.3911e-05	1.3363e-02
1	0.5	6.0653e-01	0	-2.9751e-02	4.4933e-01	1.9847e-00	3.0108e-00
	1	3.6789e-01	0	-7.8799e-02	-7.3288e-02	-4.3809e-02	1.7198e-01
	2	1.3534e-01	0	-5.7181e-03	-1.9833e-03	-3.8342e-04	1.2725e-01
1.5	0.5	6.0653e-01	0	-6.6939e-02	1.5165e-00	1.0110e01	1.2166e01
	1	3.6789e-01	0	-1.7730e-01	-2.4735e-01	-2.2927e-01	-2.8604e-01
	2	1.3534e-01	0	-1.2866e-02	-6.6935e-03	-1.9483e-03	1.1383e-01
2	0.5	6.0653e-01	0	-1.1900e-01	3.5946e-00	3.2226e01	3.6308e01
	1	3.6789e-01	0	-3.1520e-01	-5.8631e-01	-7.5776e-01	-1.2914e-00
	2	1.3534e-01	0	-2.2872e-02	-1.5866e-02	-6.1893e-03	9.0408e-02

Table 4: Variation of the velocity components and approximate velocity $\Phi_{5,u}$ with different values of r and t for cylindrical symmetry $\nu = 2$ and $\gamma = 1.4$.

t	r	ρ_0	ρ_1	ρ_2	ρ_3	ρ_4	$\Psi_{5,\rho}$
0.5	0.5	6.0653e-01	0	-1.1157e-01	-9.1967e-02	1.1104e-02	4.1404e-01
	1	3.6788e-01	6.7668e-02	1.2447e-02	-2.5938e-03	-3.0029e-03	4.4240e-01
	2	1.3534e-01	1.3737e-02	1.3168e-03	-1.1802e-05	-3.3046e-05	1.5034e-01
1	0.5	6.0653e-01	0	-4.4626e-01	7.3972e-01	2.7216e-01	-3.0729e-01
	1	3.6788e-01	1.3534e-01	4.9787e-02	-2.0162e-02	-4.8065e-02	4.8478e-01
	2	1.3534e-01	2.7474e-02	5.2674e-03	-9.2156e-05	-5.2838e-04	1.6746e-01
1.5	0.5	6.0653e-01	0	-1.0041e-00	-2.5190e-00	2.6341e-00	-2.8241e-01
	1	3.6788e-01	2.0300e-01	1.1202e-01	-6.4736e-02	-2.4435e-01	3.7382e-01
	2	1.3534e-01	4.1210e-02	1.1852e-02	-2.9832e-04	-2.6721e-03	1.8542e-01
2	0.5	6.0653e-01	0	-1.7850e-00	-6.0452e-00	1.5974e+1	8.7498e-00
	1	3.6788e-01	2.7067e-01	1.9915e-01	-1.4246e-01	-7.7978e-01	-8.4547e-02
	2	1.3534e-01	5.4947e-02	2.1069e-02	-6.6496e-04	-8.4324e-03	2.0225e-01

Table 5: Variation of the density components and approximate density $\Psi_{5,\rho}$ in flow-field with different values of r and t for cylindrical symmetry $\nu = 2$ and $\gamma = 1.4$.

t	r	p_0	p_1	p_2	p_3	p_4	$\chi_{5,p}$
0.5	0.5	-7.4377e-02	3.6089e-02	-1.0397e-02	1.1536e-02	5.8429e-03	1.0349e-01
	1	-1.6596e-02	-9.1578e-03	-4.1551e-03	-1.3764e-03	-1.2649e-04	-3.1412e-02
	2	-8.2625e-04	-2.0687e-04	-3.7133e-05	-4.6424e-06	-6.9032e-08	-1.0750e-03
1	0.5	-7.4377e-02	7.2179e-02	4.1590e-02	9.2029e-02	8.1818e-02	-1.4298e-02
	1	-1.6596e-02	-1.8316e-02	-1.6620e-02	-1.1286e-02	-2.3166e-03	-6.5135e-02
	2	-8.2625e-04	-4.1374e-04	-1.4853e-04	-3.7318e-05	-1.1332e-06	-1.4270e-03
1.5	0.5	-7.4367e-02	-1.0827e-01	-9.3577e-02	3.0914e-01	2.3992e-01	2.7284e-01
	1	-1.6596e-02	-2.7474e-02	-3.7396e-02	-3.9638e-02	-1.5046e-02	-1.3615e-01
	2	-8.2651e-04	-6.2061e-04	-3.3420e-04	-1.2695e-04	-6.0133e-06	-1.9140e-03
2	0.5	-7.4377e-02	-1.4436e-01	1.6636e-01	7.2792e-01	-3.4646e-01	-3.6269e-03
	1	-1.6596e-02	-3.6631e-02	-6.6481e-02	-9.9092e-02	-6.6579e-02	-2.8538e-01
	2	-8.2625e-04	-8.2747e-04	-5.9413e-04	-3.0425e-04	-2.0391e-05	-2.5725e-03

Table 6: Variation of the pressure components and approximate pressure $\chi_{5,p}$ in flow-field with different values of r and t for cylindrical symmetry $\nu = 2$ and $\gamma = 1.4$.

For the cylindrical symmetry, Tables 4–6 show that the contribution of higher order components in series solution for velocity, density and pressure distribution are relatively small for all the time t and $r \geq 1$. Therefore we can conclude that ratio test for convergence of series will be satisfied and solution of system of gas dynamics equations exist in form of series. The approximate velocity in cylindrical symmetry decrease for all r and $t \leq 1$ but for $t \geq 1$, it first decreases steeply and then increases with r. Tables 5–6 and Figs. 2*b*–2*c* show that there exist multiple discontinuity in distributions of density and pressure for cylindrical symmetry and their position can be identified.

t	r	u_0	u_1	u_2	u_3	u_4	$\Phi_{5,u}$
0.5	0.5	6.0653e-01	0	1.1492e-01	2.9372e-01	4.8312e-01	1.4983e-00
	1	3.6788e-01	0	-7.4681e-03	-1.8172e-03	7.5676e-04	3.5935e-01
	2	1.3534e-01	0	-1.1252e-03	-2.0685e-04	-2.4869e-05	1.3398e-01
1	0.5	6.0653e-01	0	4.5968e-01	2.3498e-00	7.2158e-00	1.06318e01
	1	3.6788e-01	0	-2.9872e-02	-1.4538e-02	1.2029e-02	3.3550e-01
	2	1.3534e-01	0	-4.5006e-03	-1.6548e-03	-3.9893e-04	1.2878e-01
1.5	0.5	6.0653e-01	0	1.0343e-00	7.9305e-00	3.2192e01	4.1763e02
	1	3.6788e-01	0	-6.7213e-02	-4.9065e-02	6.0224e-02	3.1183e-01
	2	1.3534e-01	0	-1.0126e-02	-5.5850e-03	-2.0283e-03	1.1760e-01
2	0.5	6.0653e-01	0	1.8387e-00	1.8798e01	8.2546e01	1.0379e02
	1	3.6788e-01	0	-1.1949e-01	-1.1630e-01	1.8737e-01	3.1946e-01
	2	1.3534e-01	0	-1.8002e-02	-1.3239e-02	-6.4487e-03	9.7646e-02

Table 7: Variation of the velocity components and approximate velocity $\Phi_{5,u}$ in flow-field with different values of r and t for spherical symmetry $\nu = 3$ and $\gamma = 1.4$.

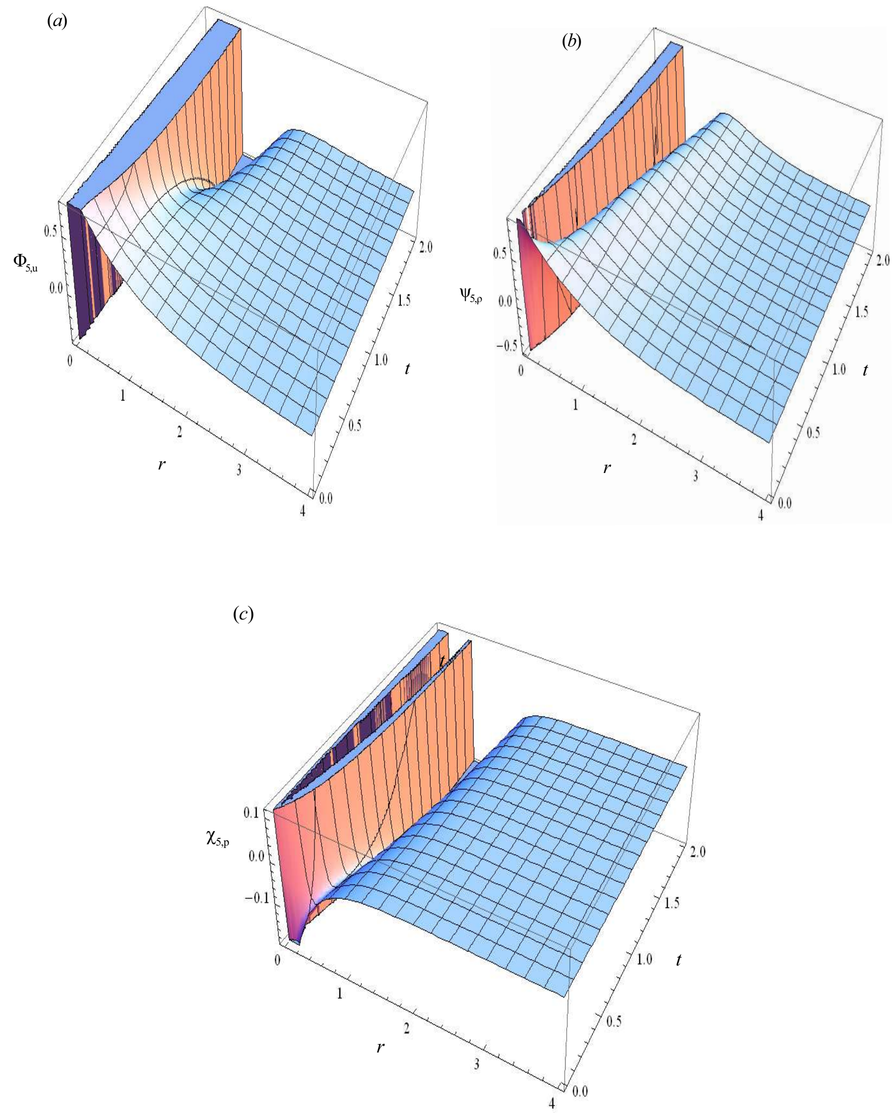

Figure 2: Distribution of flow variables: (**a**) approximate velocity (**b**) approximate density, (**c**) approximate pressure, in flow field for cylindrical symmetry

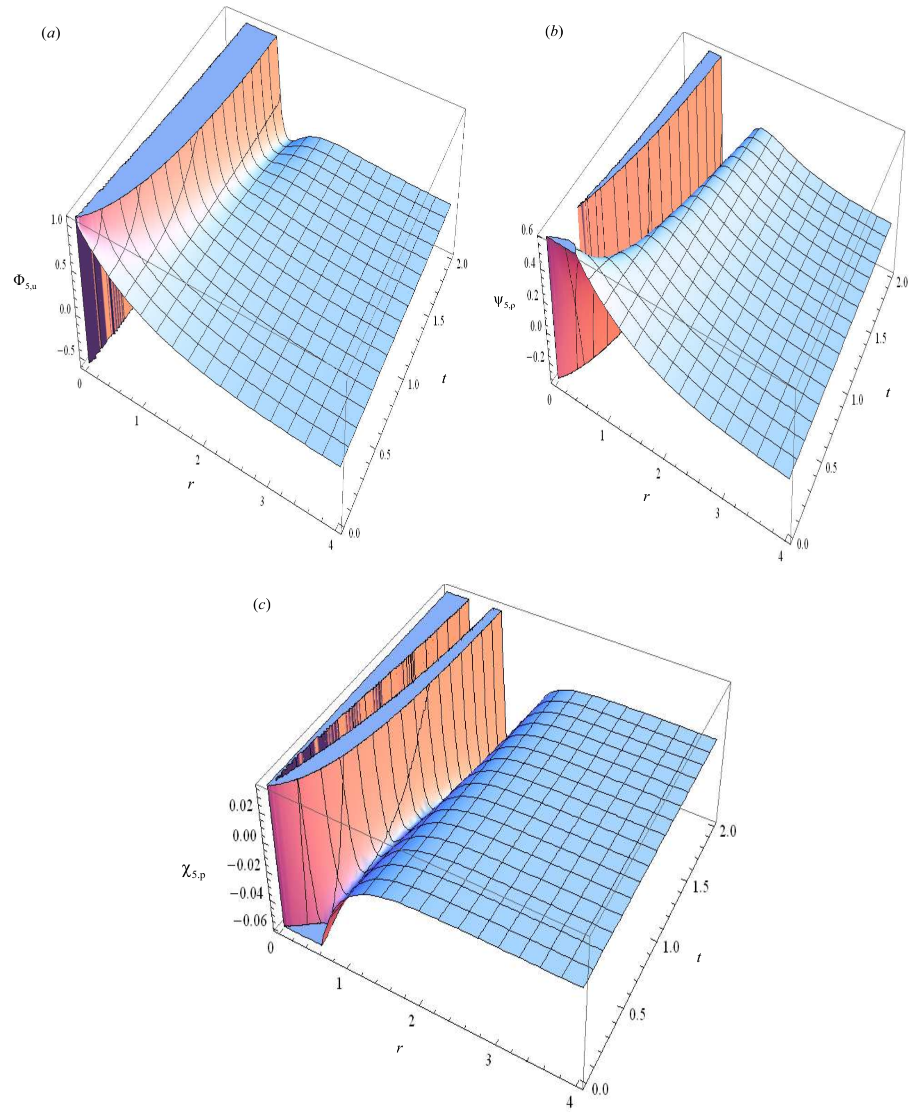

Figure 3: Distribution of flow variables: (**a**) approximate velocity, (**b**) approximate density, (**c**) approximate pressure, in flow field for spherical symmetry

t	r	ρ_0	ρ_1	ρ_2	ρ_3	ρ_4	$\Psi_{5,\rho}$
0.5	0.5	6.5603e-01	-3.6788e-01	-3.9048e-01	-1.8426e-01	6.3597e-02	-2.7249e-01
	1	3.6788e-01	0	-1.2447e-02	-6.8625e-03	-1.3355e-03	3.4724e-01
	2	1.3534e-01	9.1534e-03	7.7461e-04	-3.6891e-05	-2.5382e-05	1.4521e-01
1	0.5	6.0653e-01	-7.3576e-01	-1.5619e-00	-1.0436e-00	3.1547e-00	4.2028 e-01
	1	3.6788e-01	0	-4.9787e-02	-5.5346e-02	-2.1813e-02	2.4093e-01
	2	1.3534e-01	1.8316e-02	3.0984e-03	-2.9303e-04	-4.0587e-05	1.5605e-01
1.5	0.5	6.0653e-01	-1.1036e-00	-3.5143e-00	-1.0979e-00	2.8962e01	2.3853e01
	1	3.6788e-01	0	-1.1202e-01	-1.8930e-01	-1.1529e-01	-4.8734e-02
	2	1.3534e-01	2.7474e-02	6.9715e-03	-9.7722e-04	-2.0530e-03	1.6675e-01
2	0.5	6.0653e-01	-1.4715e-00	-6.2476e-00	5.4390e-00	4.8551e+1	4.6878e01
	1	3.6788e-01	0	-1.9915e-01	-4.5704e-01	-3.9109e-01	-6.7941e-01
	2	1.3534e-01	3.6631e-02	1.2394e-02	-2.2773e-03	-6.4823e-03	1.7560e-01

Table 8: Variation of the density components and approximate density $\Psi_{5,\rho}$ in flow-field with different values of t and r for spherical symmetry $\nu = 3$ and $\gamma = 1.4$.

t	r	p_0	p_1	p_2	p_3	p_4	$\chi_{5,p}$
0.5	0.5	-7.4377e-02	2.7067e-02	6.0469e-02	7.2436e-02	2.2298e-02	1.0790e-01
	1	-1.6595e-02	4.8842e-03	-1.6396e-03	-2.5709e-04	1.1746e-05	-2.3365e-02
	2	-8.2625e-04	-1.6773e-04	-2.9321e-05	-3.5550e-06	-7.6148e-08	-1.0269e-03
1	0.5	-7.4377e-02	5.4134e-02	2.4188e-01	5.3279e-01	-7.4559e-01	8.8374e-03
	1	-1.6596e-02	-9.7683e-03	-6.5583e-03	-2.1390e-03	1.6301e-04	-3.4898e-02
	2	-8.2625e-04	-3.5463e-04	-1.1728e-04	-2.8661e-05	-1.2533e-06	-1.3089e-03
1.5	0.5	-7.4377e-02	8.1202e-02	5.4422e-01	1.5355e-00	-1.5972e01	-1.3885e01
	1	-1.6596e-02	-1.4653e-02	-1.4756e-02	-7.6819e-03	5.4769e-04	-5.3139e-02
	2	-8.2625e-04	-5.0319e-04	-2.6389e-04	-9.7979e-05	-6.6856e-06	-1.6980e-03
2	0.5	-7.4377e-02	1.0827e-01	9.6751e-01	2.7680e-00	-1.0561e02	-1.0184e+2
	1	-1.6596e-02	-1.9537e-02	-2.6233e-02	-1.9745e-02	1.3687e-04	-8.1974e-02
	2	-8.2625e-04	-6.7093e-04	-4.6913e-04	-2.3638e-04	-2.2850e-05	-2.2255e-03

Table 9: Variation of the pressure components and approximate pressure $\chi_{5,p}$ in flow-field with different values of r and t for spherical symmetry $\nu = 3$ and $\gamma = 1.4$.

For the spherical symmetry, the tables 7–9 show that the contribution of higher order components of velocity, density and pressure in their series solution is decreasing for $r \geq 1$ and all time $t > 0$. Therefore the ratio test will be satisfied for this domain and series will converse, so the solution exists for $r \geq 1$ and all $t > 0$. The distribution of approximate velocity, approximate density and approximate pressure for spherical symmetry for different value of r and t

is shown in Tables 7–9 and Figures 3*a*–3*c*. It is seen that there exist multiple discontinuity (shock wave) in the variation of density and pressure with respect to position.

6. Conclusion

In this paper, Adomian decomposition method is successfully applied for the solution of the system of gas dynamic equations governing the motion of one-dimensional unsteady adiabatic flow of the perfect gas under the variable initial condition for all the three type of symmetry. We have obtained the distribution of approximate velocity, density and pressure directly as function of position and time which was not possible in similarity method. This paper also show the existence of multiple discontinuity or shock wave in the distribution of flow variables and identification of their positions. It is found that the solution of the system of gas dynamic equations by the Adomian decomposition method in form of the series is convergent for a domain of interest. This work show that the Adomian decomposition method can be used in study of one dimensional gas dynamic problem in all the three symmetry with more advanced and realistic gas dynamics problems.

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