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UNSTEADY MHD CONVECTION FLOW AND HEAT TRANSFER PAST A VERTICAL INCLINED PLATE IN A POROUS MEDIUM WITH VARIABLE PLATE TEMPERATURE WITH SUCTION IN A SLIP FLOW REGIME

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Abstract: The objective of the present study is to investigate the unsteady MHD free convective flow and heat transfer past an inclined plate with cosinusoidally flactuating temperature with suction in a boundary layer slip flow regime. The non-linear coupled partial differential equations are solved by pertubation technique about a small pertubation parameter ϵ. By employing the perturbation approach the solutions are obtained for velocity, temperature, skin friction and Nusselt number in terms of parameters like Prandtl number (P_r), magnetic parameter (M), plate inclination angle (α), thermal Grashof number(G_T), suction parameter (A) and rarefaction parameter (k_p) and are discussed through graphical and tabular representations. It is noted that the effect of the rarefaction parameter, inclination angle and Prandtl number is to decrease the skin friction. It is also noted that the velocity decreases with increasing inclination angle.

AMS Subject Classification: 65N06, 65N15, 65M06, 76S05
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1. Introduction

The study of Magneto hydrodynamic free convective flow and heat transfer in a porous medium has applications in engineering. Convective flows play a very

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crucial role in chemical engineering, aerospace technology, turbo machinery, cooling of nuclear reactors and MHD power generators. In addition Magneto hydrodynamics also play an important role in agriculture and petroleum industries.

The study of magneto hydrodynamics has attracted a large number of scientists due to its wide range of applications. Extensive research work on MHD convective flow past a vertical plate has been studied by Chamkha and Aly [6], Sahoo et al. [15], Das et al. [11], Das et al. [7], Das et al. [8], Aldoss et al. [3], Deka and Das [10], Ibrahim et al. [12]. Das et al. [9] analyzed hydromagnetic flow past a vertical porous plate with suction and heat source.

When the fluid particles adjacent to a plate surface do not move with the same velocity of the surface but instead slips with a tangential velocity then the flow is said to be a “slip flow”. Slip flows occurs in many practical scenarios and its effect cannot be ignored. The study of the effect of slip flow has been investigated by many authors, who include among others (Sharma [16], Sahin [14], Sharma, Chaudhary and Jha [17] and Pal Talukdar [13]). Pal Talukdar [13] investigated perturbation analysis of unsteady magnetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction.

The present study differs from that of Das et al. [9] in that heat source is ignored, plate is inclined and boundary layer is slip flow. It also differs from Pal Talukdar [13] in that the effect of thermal diffusion, heat source and chemical reaction has been ignored. In the current study the fluid flow passes over an inclined plate with cosinusoidally varying plate temperature. The study of MHD free convective flow over an inclined plate was investigated by a number of authors. Alam et al. [1] investigated the effects of variable suction and thermophoresis on steady MHD combined free-forced convective heat and mass transfer flow over a semi-infinite permeable inclined plate in the presence of radiation. In the subsequent year Alam et al. [2] studied the transient MHD free convective heat and mass transfer flow with thermophoresis past a radiate inclined permeable plate in the presence of temperature dependent viscosity. Later Uddin [18] analyzed steady convective flow of micro-polar fluids along an inclined flat plate with variable electric conductivity and uniform surface heat flux. Lately Buzuzi et al. [4] investigated the effects of various parameters on an MHD flow over an inclined plate using fitted numerical methods.

The rest of the paper is organized as follows. In Section 2 we present the model formulation. Section 3 is concerned with the method of solution. Section 4 deals with the results and their discussion. Finally, Section 5 presents concluding remarks.

2. Mathematical analysis

We consider a two dimensional unsteady flow of a laminar, viscous, incompressible electrically conducting fluid past an inclined permeable plate subjected to a transverse magnetic field. Let the x^*-axis be directed along the porous plate and the y^*-axis be normal to the plate. Then the magnetohydrodynamic unsteady convective boundary layer equations of mass, momentum and energy read

$$\frac{\partial v^*}{\partial y^*} = 0, \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + v^*\frac{\partial u^*}{\partial y^*} = \nu\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma {B_o}^2}{\rho^*}u^* + gB_T(T^* - T^*_\infty)\cos\alpha, \tag{2}$$

$$\frac{\partial T}{\partial t^*} + v^*\frac{\partial T}{\partial y^*} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^{*2}}, \tag{3}$$

where u^* and v^* are the components of dimensional velocities along the x^* and y^* directions respectively, ρ is the fluid density, and ν the kinematic viscosity, σ is the electrical conductivity of the fluid, B_o is the magnetic induction, α is plate angle of inclination, T is the temperature of the fluid in the boundary layer, g is gravitational acceleration.

It is assumed that the temperature T fluctuates cosinusoidally with time. Given the above assumptions, the appropriate boundary conditions for the velocity and temperature are

$$u^* = L^*\frac{\partial u^*}{\partial y^*}, \quad T^* = T_w + \epsilon(T_w - T_\infty)\cos n^*t^*, \quad \text{on} \quad y^* = 0,$$

$$u^* \to U^*_\infty = U_0(1 + \epsilon e^{n^*t^*}), \qquad T \to T_\infty, \quad \text{as} \qquad y^* \to \infty, \tag{4}$$

where T_w are wall dimensional temperature. T_∞ are the free stream dimensional temperature and U_o and n^* are the scale of free stream velocity and a scalar constant respectively. From the continuity equation (1) it is deduced that the suction velocity is a function of time only and therefore we assume that it takes the following form:

$$V^* = -V_0(1 + \epsilon Ae^{n^*t^*}) \tag{5}$$

where V_0 is the scale of function velocity and A and ϵ are small such that

$\epsilon << 1$ and $\epsilon A << 1$. We introduce the following dimensionless quantities:

$$y = \frac{V_0 y^*}{v}, \quad t = \frac{V_0^2 t^*}{v}, \quad n = \frac{n^* \nu}{V_0^2}, \quad \nu = \frac{\mu}{\rho},$$

$$T = \tfrac{T^* - T^*_\infty}{T^*_W - T^*_\infty}, \quad u = \tfrac{u^*}{U_0}, \quad k_p = \tfrac{L^* V_0}{nu}, \quad G_T = \nu \beta^*_T g \tfrac{(T^*_w - T^*_\infty)\cos\alpha}{U_0 V_0^2}, \tag{6}$$

$$M = \tfrac{\sigma \nu B_0^2}{\rho V_0^2}, \quad P_r = \tfrac{\mu C_p}{\kappa},$$

where P_r is the Prandtl number, G_T is the thermal Grashof number, M is the magnetic field parameter and μ is the viscosity of fluid. In view of equations (5)-(6) the non-dimensional form the governing equations (1)-(3) read:

$$\frac{\partial u}{\partial t} - (1 + \epsilon A e^{nt}) \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G_T T \cos\psi - M u, \tag{7}$$

$$\frac{\partial T}{\partial t} - (1 + \epsilon A e^{nt}) \frac{\partial T}{\partial y} = \frac{1}{P_r} \frac{\partial^2 T}{\partial y^2}, \tag{8}$$

and the associated boundary conditions (4) is given by the following dimensionless form:

$$\begin{aligned} &u = k_p \tfrac{\partial u}{\partial y}, \quad T = 1 + \epsilon \cos nt \quad \text{at} \quad y = 0, \\ &u = 1 + \epsilon e^{nt}, \quad T \to 0 \text{ as } \quad y \to \infty. \end{aligned} \tag{9}$$

3. Analytical method of solution

The basic governing equations (7)-(8) are pde's that cannot be solved in closed form. In order to solve them analytically we reduce the system of pde's to a system of ode's in dimensionless form by representing the linear velocity, and temperature as

$$\begin{aligned} u(y,t) &= u_0(y) + \epsilon e^{nt} u_1(y) + O(\epsilon^2) + \ldots, \\ T(y,t) &= T_0(y) + \epsilon e^{nt} T_1(y) + O(\epsilon^2) + \ldots. \end{aligned} \tag{10}$$

Substituting (10) in (7)-(8), and equating the harmonic and non-harmonic terms ignoring terms of $O(\epsilon^2)$ gives the following equations for u_0, T_0 and u_1, T_1,

$$u_0'' + u_0' - M u_0 = -G_T T \cos\alpha, \tag{11}$$

$$T_0'' + P_r T_0' = 0, \tag{12}$$

subject to the boundary conditions

$$u_0 = k_p u_0', \quad T_0 = 1, \quad \text{on} \quad y = 0,$$

$$u_0 = 1, \quad T_0 \to 0, \quad \text{as} \quad y \to \infty, \tag{13}$$

and

$$u_1'' + u_1' - (n+M)u_1 = -G_T T_1 \cos\alpha - A u_0', \tag{14}$$

$$T_1'' + P_r T_1' - n P_r T_1 = -A P_r T_0', \tag{15}$$

subject to the boundary conditions

$$u_1 = k_p u_1', \quad T_1 = 1 \quad \text{on} \quad y = 0,$$

$$u_1 = 1, \quad T_1 \to 0 \quad \text{as} \quad y \to \infty, \tag{16}$$

where the prime denotes ordinary differentiation with respect to y.

The solution of the equations (11)-(12) and (14)-(15) subject to the boundary conditions (13) and (16) are:

$$u_0 = a_3 e^{-k_1 y} + a_5 e^{-P_r y}, \tag{17}$$

$$u_1 = a_9 e^{-k_5 y} + a_{11} e^{-k_3 y} + a_{12} e^{-P_r y} + a_{13} e^{-k_1 y}, \tag{18}$$

$$T_0 = e^{-P_r y}, \tag{19}$$

$$T_1 = (1 - a_8) e^{-k_3 y} + a_8 e^{-P_r y}, \tag{20}$$

where

$$k_1 = \frac{1 + \sqrt{1 + 4M}}{2}, \quad k_3 = \frac{P_r}{2}\left(1 + \sqrt{1 + \frac{4n}{P_r}}\right),$$

$$k_5 = \frac{1 + \sqrt{1 + 4(n+M)}}{2}, \quad a_{11} = \frac{G_T \cos\alpha(1 - a_8}{k_3^2 - k_3 - (n+M)},$$

$$a_9 = \frac{(k_3^2 k_p - 1)a_{11} + (P_r^2 k_p - 1)a_{12} + (k_1^2 k_p - 1)a_{13}}{1 - k_p k_5^2},$$

$$a_{12} = \frac{-a_8 G_T \cos\alpha - A a_5}{P_r^2 - P_r - (n+M)}, \quad a_{13} = \frac{-A a_3}{k_1^2 - k_1 - (n+M)}.$$

By considering (10) the final form of the velocity and temperature distributions in the boundary layer are:

$$u(y,t) = a_3 e^{-k_1 y} + a_5 e^{-P_r y} + \epsilon e^{nt}\Big(a_9 e^{-k_5 y} + a_{11} e^{-k_3 y} + a_{12} e^{-P_r y} + a_{13} e^{-k_1 y}\Big), \quad (21)$$

$$T(y,t) = e^{-P_r y} + \epsilon e^{nt}\Big((1-a_8)e^{-k_3 y} + a_8 e^{-P_r y}\Big). \quad (22)$$

The skin friction coefficient C_f and the Nusselt number Nu at the wall of the plate are defined as follows:

$$C_f = -[a_3 k_1 + a_5 P_r + \epsilon e^{nt}\left(a_9 k_5 + a_{11} k_3 + a_{12} P_r + a_{13} k_1\right)], \quad (23)$$

$$\frac{Nu}{Re} = \frac{\partial\theta}{\partial y}|_{y=0} = -[P_r + \epsilon e^{nt}\left((1-a_8)k_3 + a_8 P_r\right)], \quad (24)$$

where Re is the local Reynold's number.

4. Results and Discussion

In the current study we have investigated unsteady MHD convective flow and heat transfer past an inclined plate in a porous medium with cosinusoidal flactuating temperature with suction in a slip flow regime. Computed results of the study are presented graphically in Figures 1 - 9 and in tabular form in Tables 1 - 4. The following flow conditions were fixed in constructing tables and plotting figures unless specified, $\epsilon = 0.2$, $nt = \frac{\pi}{2}$, $G_T = 1, M = 1$; $P_r = 1$, $\alpha = 0$, $K_p = 0.1$, $A = 0.5$, $t = 1$.

The effect of G_T, M, α, n and k_p on velocity are shown in Figures 1 - 6, respectively. Figure 1 shows the thermal Grashof number has the effect of increasing the velocity profile. Figure 2 displays the effect of the magnetic parameter on the velocity profile. It is observed that close to the wall the magnetic parameter enhances the velocity profile. However for $y \geq 1$ the effect is reversed and the presence of magnetic field tend to retard fluid flow.

The effect of plate inclination angle to the velocity profile is depicted in Figure 3. It is observed that the fluid velocity is maximum when the plate is vertical ($\alpha = 0$) and least when plate is horizontal ($\alpha = \pi/2$). In other words the plate inclination angle α has the effect of decreasing the buoyancy force. It is maximum when the plate is vertical and is reduced to zero when the plate

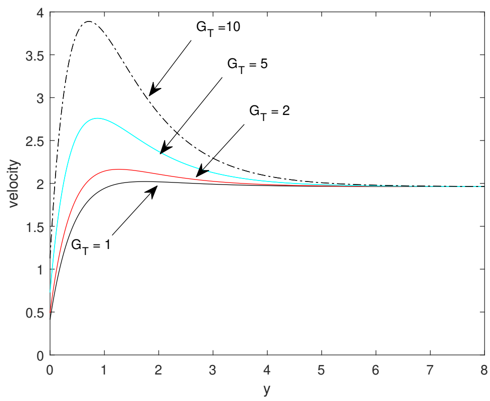

Figure 1: Velocity profile for different values of the thermal Grashof number G_T

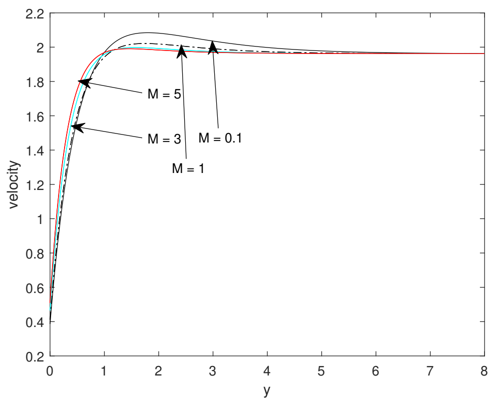

Figure 2: Velocity profile for varying magnetic parameter M

is horizontal. Figure 4 shows the effect of varying angular frequency n, on the velocity profile. It is noticed that increasing angular frequency enhances the velocity profile. The effect of suction on velocity is depicted in Figure 5. It is observed that suction enhances velocity for $y < 1$ and is reversed for $y \geq 1$. The effect of rarefaction parameter k_p on the velocity profile is illustrated in Figure 6. It is noticed that the rarefaction parameter has the effect of increasing the velocity profile.

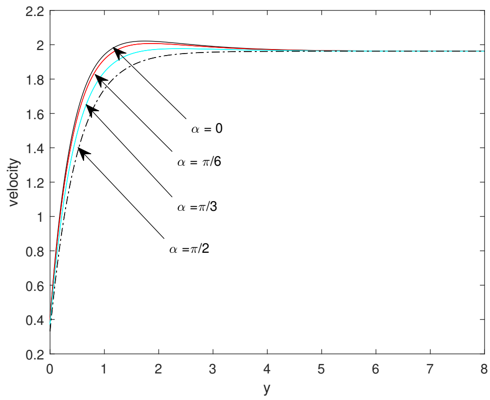

Figure 3: Velocity profile for varying angle of inclination α

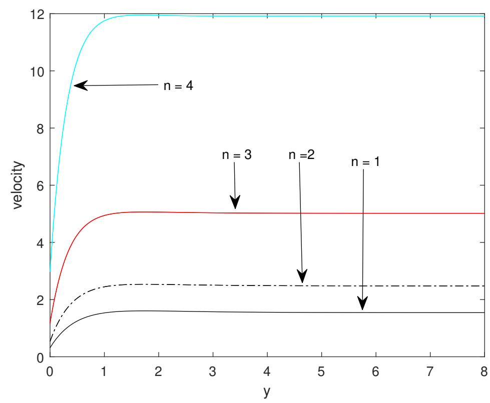

Figure 4: Velocity profile for varying angular frequency n

The effect of P_r, n and A on the temperature profiles is depicted in Figures 7 - 9, respectively. Figure 7 shows that the temperature profile across the boundary layer for different values of the Prandtl number P_r. It is observed that that increasing the Prandtl number has the effect of decreasing the temperature of the flow field. In Figure 8 we analyze the effect of angular frequency n on the temperature of the flow field. it is shown that the angular frequency enhances the temperature of the flow field. Figure 9 illustrates the effect of varying the suction parameter A on the temperature of the flow field. The observation is

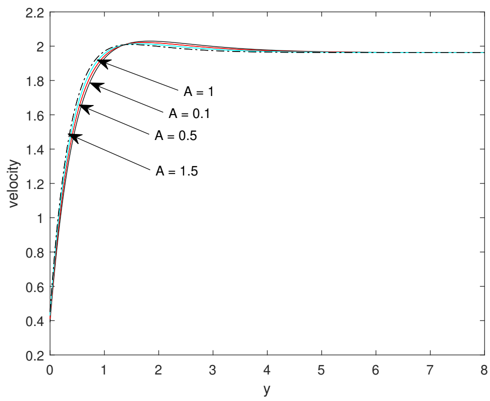

Figure 5: Velocity profile for varying suction parameter A

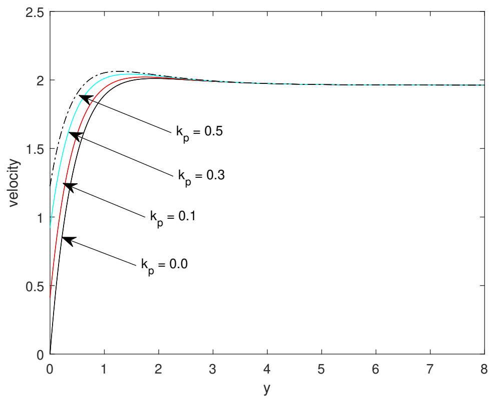

Figure 6: Velocity profile for varying rarefaction parameter k_p

that increasing the suction parameter reduces the temperature of the flow field.

The effects of magnetic parameter M, angular frequency n and suction parameter A, rarefaction parameter k_p, Prandtl number P_r and inclination angle α on the skin friction C_f and heat flux Nu is depicted in Tables 1 - 4. From Table 1 it is observed that the increase in the rarefaction parameter k_p has the effect of decreasing the skin friction has no effect on both the Nusselt number. The skin friction increases as the magnetic parameter increases. From Table 2 it is noticed that as the Prandtl number increases the skin friction

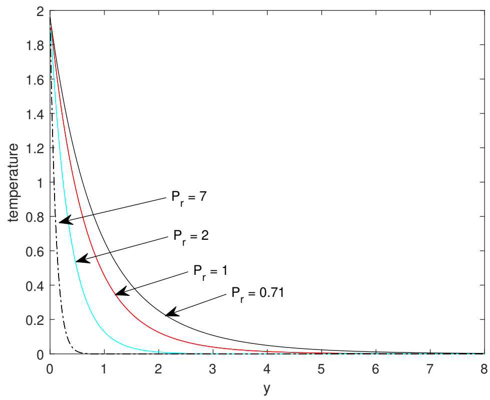

Figure 7: Temperature profile for varying Prandtl numbers P_r

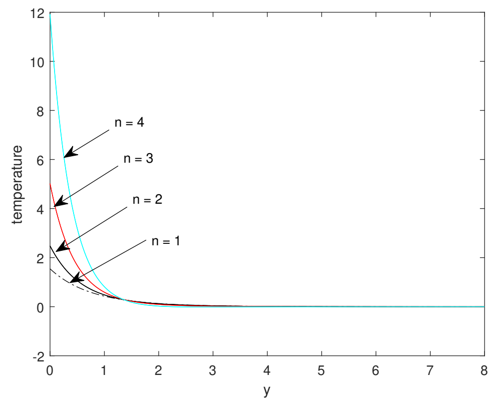

Figure 8: Temperature profile for varying angular frequency n

decreases. It is also observed that the skin friction increases as the angular frequency increases. From Table 3 it is shown that as the inclination angle α has a decreasing effect on the skin friction. It is also noticed that suction has enhancing effect on the skin friction. Table 4 illustrates that the Prandtl number has an increasing effect on the heat flux. It is also observed that the heat flux is enhanced by the angular frequency.

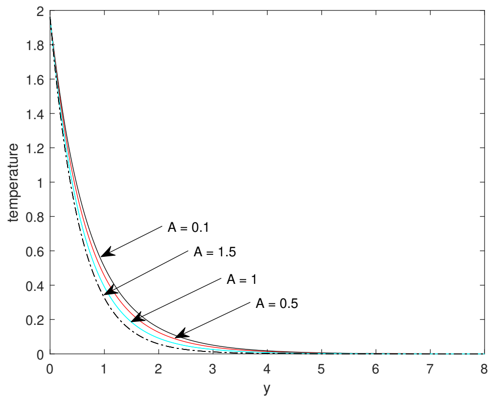

Figure 9: Temperature profile for varying suction parameter A

Table 1: Variation in the value of the skin friction at the wall against k_p for different values of M

k_p	$M = 0.1$	$M = 0.5$	$M = 1$
0.0	36.0482	37.0154	38.2754
0.1	28.5370	29.0279	29.7097
0.3	20.1138	20.2394	20.4829
0.5	15.5148	15.5179	15.6113

5. Concluding remarks

In this paper investigations were carried out on the effects of thermal Grashof number, magnetic parameter, plate inclination angle and suction on the unsteady MHD convective flow and heat transfer past an inclined plate in a porous medium with co-sinusoidal fluctuating temperature with suction in a slip flow regime. The governing equations were transformed into a system of nonlinear differential equations by a perturbation technique. The following conclusions are drawn from the present numerical study:

1. Velocity increases with increase in the suction parameter and magnetic parameter for $y < 1$. Furthermore velocity increases with increase in the thermal Grashof number, angular frequency and rarefaction parameter.

2. Velocity decreases with increase in the suction parameter and the magnetic parameter for $y \geq 1$. Velocity also decreases with increasing the inclination angle.

Table 2: Variation in the value of the skin friction at the wall against P_r for different values of n

P_r	$n = 1$	$n = 2$	$n = 3$
0.71	2.0120	3.2336	6.6046
1	1.9156	3.1171	6.4404
3	1.6744	2.8216	6.0173
7	1.5736	2.6950	5.8292

Table 3: Variation in the value of the skin friction at the wall against α for different values of A

α	$A = 0.1$	$A = 0.5$	$A = 1$
0.0	2.4034	2.4510	2.5105
$\pi/6$	2.3376	2.3868	2.4482
$\pi/3$	2.1580	2.2112	2.2778
$\pi/2$	1.9126	1.9714	2.0450

3. Temperature increases with increasing angular frequency.
4. Temperature falls with increase in the Prandtl number and suction parameter.
5. Skin friction increases with increase in the magnetic parameter, angular frequency and suction. On the other hand skin friction decreases with increase in the magnetic parameter, Prandtl number, inclination angle and rarefaction parameter.
6. The Nusselt number increases with increase in the Prandtl number and the angular frequency.

Most of the above findings agrees with previous findings except for the following:

(i) Magnetic parameter enhances velocity near the wall.

(ii) Suction enhances velocity near the wall.

(iii) Magnetic parameter enhances the skin friction.

Table 4: Variation in the value of the heat flux Nu at the wall against P_r for different values of n

P_r	$n = 1$	$n = 2$	$n = 3$
0.71	-1.5080	-3.3052	-8.7150
1	-2.0477	-4.3251	-11.1227
2	-3.8833	-7.6956	-18.8491
7	-12.9768	-23.9425	-54.8449

References

[1] M.S. Alam, M.M. Rahman and A. Satter, Effects of variable suction and thermophoresis on steady MHD combined free-forced convective heat and mass transfer flow over a semi-infinite permeable inclined plate in the presence of thermal radiation, *Int. J. Therm. Sci.*, **47**, No 6 (2008), 758-765.

[2] M.S. Alam, M.M. Rahman and A. Satter, Transient magnetohydrodynamic free convective heat and mass transfer flow with themophoresis past a radiate inclined permeable plate in the presence of variable chemical reaction and temperature dependent viscosity, *Nonlinear Analysis: Model Control*, **14** (2009), 3-20.

[3] T.K. Aldoss, M.A. Al-Nimr and A.F. Khadrawi, Effect of the local acceleration term on the MHD transient free convection flow over a vertical plate, *International J. for Numerical Methods in Heat & Fluid Flow*, **15**, No 3 (2005), 296-305.

[4] G. Buzuzi, J.B. Munyakazi and K.C. Patidar, A fitted numerical method to investigate the effect of various parameters on an MHD flow over an inclined plate, *Numerical Methods for Partial Differential Equations*, (2015), 107-120; DOI 10.1002/num.

[5] A.J. Chamkha, MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction, *Int. Comm. Heat Mass Transfer*, **30** (2003), 413-422.

[6] A.J. Chamkha and A.M. Aly, MHD free convection flow of a nanofluid past a vertical plate in the presence of heat generation or absorption effects, *Chemical Engineering Communications*, **198**, No 3 (2010), 425-441.

[7] S.S. Das, S.K. Sahoo and G.C. Das, Numerical solution of mass transfer effects on unsteady flow past an accelerated vertical porous plate with suction, *Bull. Malays. Math. Sci. Phys. Sci., Soc.*, (2006), 33-42.

[8] S.S. Das, A. Satapathy, J.K. Das and S.K. Sahoo, Numerical solution of unsteady free convective MHD flow past an accelerated vertical plate with suction and heat flux, *J. Utra Sci. Phys. Sci.*, 19, No 1 (2007), 105-112.

[9] S.S. Das, U.K. Tripathy and J.K. Das, Hydromagnetic convective flow past a vertical porous plate through a porous medium with suction and heat source, *International J. of Energy and Environment*, **1**, No 3 (2010), 467-478.

[10] R. Deka and U.N. Das, Effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction, *Forsch Ingenieurwes*, **60** (1994), 284-287.

[11] U.N. Das, R. Deka and V.M. Soundalgekar, Effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction, *Forschung im Ingenieurwesen*, **60**, No 10 (1994), 284-287.

[12] F.S. Ibrahim, A.M. Elaiw and A.A. Bakr, Effect of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi infinite vertical permeable moving plate with heat source and suction, *Commun. Nonlinear Sci. Numer. Simulat.*, **13** (2008), 1056-1066.

[13] D. Pal and P. Talukdar, Pertubation analysis of unsteady magetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction , *Commun. Nonlinear Sci. Numer. Simulat.*, **15** (2010), 1813-1830.

[14] A. Sahin, Influence of a chemical reaction on transient Mhd free convective flow over a vertical plate in slip-flow regime, *Emirates J. for Engineering Research*, **15**, No 1 (2010), 25-34.

[15] P.K. Sahoo and S. Biswal, Magnetohydrodynamic unsteady free convection flow past an infinite vertical plate with constant suction and heat sink, *Ind. J. Pure Appl. Math.*, **34**, No 1 (2003), 145-155.

[16] P.K. Sharma, Flactuating thermal and mass diffusion on unsteady free convective flow past a vertical plate in slip-flow regime, *Latin American Applied Research*, **35** (2005), 313-319.

[17] P.K. Sharma and R.C. Chaudhary, Effect of variable suction on transient free convection viscous incompressible flow past a vertical plate with periodic temperature variation in slip-flow regime, *Emirates J. for Egineering Research*, **8** (2003), 33-38.

[18] M.J. Uddin, Convective flow of micropolar fluids along an inclined flat plate with variable electric conductivity and uniform surface heat flux, *DAFFODIL International University J. of Science and Technology*, **6**, No 1 (2011), 69-79.