

**AN APPLICATION OF FRACTIONAL CALCULUS TO  
GEOPHYSICS: EFFECT OF A STRIKE-SLIP FAULT ON  
DISPLACEMENT, STRESSES AND STRAINS IN  
A FRACTIONAL ORDER MAXWELL TYPE  
VISCO-ELASTIC HALF SPACE**

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**Abstract:** In this paper a creeping movement across a very long, strike-slip fault vertical to the free surface and of finite width is considered in an isotropic, homogeneous, visco-elastic fractional order Maxwell type half space. A mathematical model for such fault movement is developed during the period when there is no fault movement and also for the aseismic period which is restored after the creeping movement. The analytical expressions of displacement, stresses and strains for both the period are determined by the use of Green's function technique and correspondence principle in terms of Mittag-Leffler function. Finally these displacement, stresses and strains are numerically computed with suitable values of the model parameters and the results thus obtained are presented graphically. A detailed study of these expressions can focus some light on the nature of the stress accumulation near the fault and the study of such earthquake fault dynamical models helps us to understand mechanism of the lithosphere-asthenosphere system.

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## 1. Introduction

There has been an increasing attentiveness in the use of static or quasistatic displacement, stresses and strains for the inspection of earthquake occurrence. One of the main consideration of seismologists and geological engineers is the modelling of dynamic procedure caused on an earthquake. The occurrence of earthquake can be predict by using machine learning techniques P. Debnath et al. [1]. It has been observe that two successive seismic occurrence in a seismically alert zone are generally divide by a long aseismic interval in the course of which slow and steady aseismic surface fluctuation are detected with the help of ultra-modern measuring apparatus such as strain meter, tilt meter etc. Afore-said aseismic surface fluctuation designate that slow aseismic change in stress and strain are occurring in the region which may ultimately accelerate sudden or creeping movements across the seismic faults. These faults conceivably strike-slip or dip -slip type, finite or long , surface breaking or buried situating in the region. To understand the mechanism of earthquake processes it is necessary to develop mathematical models to study the small ground deformation observed during the aseismic period in the seismically active regions.

An introducing work including static ground deformation in elastic media was instigate by J.A. Steketee [2],[3], M.A. Chinnery [4]- [6], T. Maruyama [7], [8]. D.L. Turcotte et al. [9], J.C. Savage [10] did remarkable works in analysing the displacement, stress and strain for strike-slip movement of the fault in the elastic medium. Later a few theoretical models have been expanded by K. Rybycki [11], A. Mukhopadhyay [12], A. Mukhopadhya et al. [13]. U. Ghosh et al. [14], P. Segall [15], S. Sen et al. [16] did wonderful works in analysing the displacement, stresses and strains in the layered medium. S. Sen et al.[17] and P. Debnath et al.[18],[19] discussed about long interacting strike-slip faults in the viscoelastic half space. There after the models for finite strike-slip fault and infinite dip-slip fault under tectonic forces were developed by P. Debnath et al. [20], [21] and D. Mondal et al. [22], respectively. In most of the cases elastic or visco-elastic half space of Maxwell type and Standard linear solid or layered medium were considered to represent the lithosphere -asthenosphere system. To the best of our knowledge, no theoretical model has still been developed in the visco-elastic fractional order Maxwell type half

space to represent earthquake faults. The study of [23]-[26] and observations in the seismically active regions during aseismic period suggest that fractional order Maxwell type visco-elastic material may be a suitable representation of the lithosphere-asthenosphere system. This paper is therefore an application of fractional calculus to study the earthquake faults in lithosphere - asthenosphere system.

## 2. Formulation

A two-dimensional theoretical model with a long vertical surface breaking strike-slip fault  $F$  of width  $D$  is taken in the lithosphere- asthenosphere system consisting of a visco-elastic half space of fractional order Maxwell type material. To represent this, we instigate a rectangular cartesian co-ordinate technique  $(y_1, y_2, y_3)$  with  $y_3 = 0$  is the plane of free surface,  $y_3$  axis referring downwards such that the visco-elastic half space can be detail by  $y_3 \geq 0$ . The fault  $F$  is taken in the half space  $y_3 \geq 0$  with its upper edge on the free surface along which  $y_1$  axis is directed and  $y_2$  axis is perpendicular to  $y_1$  axis, lying on the free surface so that the plane of the fault is given by  $y_2 = 0$ . The length of the fault is presume to be elongate differentiate to its width  $D$  such that the components of displacement  $(u, v, w)$ , stresses  $\tau_{ij}$ , and strain  $e_{ij}, i, j = 1, 2, 3$  are independent of  $y_1$  and are functions of  $y_2, y_3$  and time  $t$  only and they separate out in two distinct and mutually independent groups - one group containing the components  $u, (\tau_{12}, \tau_{13}), (e_{12}, e_{13})$  corresponding with the strike- slip movement and the additional group including the enduring units accompanied with a conceivable dip-slip movement of the fault. We here think about the strike-slip movement over the fault.

The constitutive laws provide the relation between stress and strain possibly including time derivatives. We here consider strike -slip movement across the fault when the medium is in aseismic state ( $t = 0$ ) for which the displacement  $u$ , stresses  $\tau_{12}, \tau_{13}$  and strains  $E_{12}, E_{13}$  are present. The stress-strain relations for fractional order Maxwell model of visco-elastic material are taken as follows (M.A. Matlob and Y. Jamali [26]):

$$\left(\frac{1}{\eta} + \frac{1}{\mu} \frac{\partial^\alpha}{\partial t^\alpha}\right) \tau_{12} = \frac{\partial^\alpha e_{12}}{\partial t^\alpha}, \quad (1)$$

$$\left(\frac{1}{\eta} + \frac{1}{\mu} \frac{\partial^\alpha}{\partial t^\alpha}\right) \tau_{13} = \frac{\partial^\alpha e_{13}}{\partial t^\alpha}, \quad (2)$$

where the operator  $\frac{\partial^\alpha}{\partial t^\alpha}$  is the fractional order derivative operator, as defined by several authors in fractional calculus. But the most widely used one is the

Caputo derivative of fractional order  $\alpha$  ( $n - 1 \leq \alpha \leq n$ ) of a function  $f(t)$ , introduced M. Caputo in 1967, [27] and defined as follows:

$${}_a^C D_t^\alpha f(t) = \frac{1}{\Gamma(n - \alpha)} \int_a^t \frac{D^n f(\zeta)}{(t - \zeta)^{\alpha - n + 1}} d\zeta.$$

Here  $D^n$  is the  $n^{th}$  derivative operation and  $C$  is used to honor Caputo,  $\alpha$  is the fractional order,  $a$  is the lower limit of integration, respectively. In equations (1) and (2),  $\eta$  is the effective viscosity and  $\mu$  is the effective rigidity of the materials.

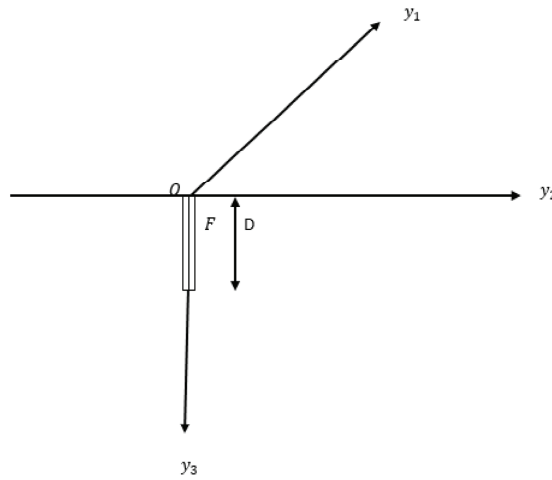


Figure 1: Section of the model by the plane  $y_1 = 0$ .

For the small deformations, if the inertial forces are very small so that the acceleration can be taken to be negligible and if there is no body forces acting in the system during our consideration, then the quasi-static equilibrium equation is

$$\frac{\partial}{\partial y_2}(\tau_{12}) + \frac{\partial}{\partial y_3}(\tau_{13}) = 0, \text{ where } (-\infty < y_2 < \infty, y_3 \geq 0, t \geq 0). \quad (3)$$

For the fault  $F$ ,

$$\left. \begin{aligned} &\tau_{12}(y_2, y_3, t) \rightarrow \tau_{\infty}(t) = \tau_{\infty}(0)(1 + kt), \\ &\quad \text{as } |y_2| \rightarrow \infty, \quad (y_3 \geq 0, t \geq 0) \\ &\quad \text{and } (\tau_{12})_0(y_2, y_3, 0) \rightarrow \tau_{\infty}(0) \\ &\quad \text{as } |y_2| \rightarrow \infty, \quad (y_3 \geq 0, t \geq 0). \\ \text{On } y_3 = 0, \quad \tau_{13}(y_2, y_3, t) = 0 \quad &\text{as } (|y_2| \rightarrow \infty, \quad t \geq 0) \\ \text{also } \quad \tau_{13} \rightarrow 0 \quad \text{as } y_3 \rightarrow \infty, \quad &(|y_2| \rightarrow \infty, \quad t \geq 0) \end{aligned} \right\}, \quad (4)$$

where  $\tau_{\infty}(t) = \tau_{\infty}(0)(1 + kt)$  where  $k > 0$  and small, is the value of  $\tau_{12}$  far away from the fault maintained by slow linearly increasing tectonic forces and vary with time. Let  $u_0, (\tau_{12})_0, (\tau_{13})_0, (e_{12})_0, (e_{13})_0$  are the value of  $u, \tau_{12}, \tau_{13}, e_{12}, e_{13}$  at time  $t = 0$ . They are the function of  $y_2, y_3$  and satisfy the relation (1) to (4).

### 3. Solution

Partially differentiating (1) with respect to  $y_2$  and (2) with respect to  $y_3$ , after that we adding and from (3) with initial condition, we get

$$\nabla^2 U = 0, \quad \text{where } U = u - u_0. \quad (5)$$

Taking Laplace transform of the resulting equation with respect to time  $t$ , we get

$$\nabla^2 \bar{U} = 0, \quad \text{where } \bar{U} = \bar{u} - \frac{u_0}{s}, \quad (6)$$

where  $s$  is the Laplace Transform variable.

Throughout the system, the displacement, stresses, and strains are all continuous and all the equations from (1) to (4) are valid. Tectonic forces far away from the fault due to mantle convection in the lithosphere-asthenosphere system cause the fault to creep leading to an earthquake. For the fault  $F$ ,  $\tau_{12}$  increases gradually with time and finally tends to  $\tau_{\infty}(t)$  but we suppose that the geological state in addition the feature of the fault  $F$  is such that it slips when the magnitude of stress  $\tau_{12}$  extended some critical value  $\tau_c$  (say)  $< \tau_{\infty}$  after time  $T$  (say). Here we assume  $\tau_c = 200$  bar i.e.,  $2 \times 10^7 \text{ N/m}^2$  (Pascal) and it is observed that  $\tau_{12}$  extend the value 200 bar after time  $T = 114$  years.

Let, after a time  $t = T$  the accumulated stress  $\tau_{12}$  near  $F$  transcend the critical level  $\tau_c$  and the fault begin the creeping movement. The accumulated stress will let out at a minimum relatively and the fault becomes locked again

when the shear stress near it has acceptably been let out. We eliminate this short period of time during and instantly after creeping movement and assume the model after the re-establishment of the aseismic state, which occurs when the seismic distraction near the fault moderately vanish.

For  $t > T$ , all the basic equation (1) to (6) remain valid and are continuous everywhere except for the fault  $F$  across which the displacement component  $u$  has a discontinuity which characterizes the creeping fault movement across  $F$  given by  $[u]_F = U'(t_1)f(y_3)H(t_1)$  across  $F(y_2 = 0, 0 \leq y_3 \leq D, t_1 = t - T > 0)$ , where  $[u]_F$  is the discontinuity in displacement across  $F$  and  $H(t_1)$  is the Heaviside unit step function. Creep velocity  $v(t_1) = \frac{\partial U'(t_1)}{\partial t_1}$  and  $v(t_1), U'(t_1)$  vanishes for  $t_1 \leq 0$ . Taking Laplace transform of displacement discontinuity, then

$$\overline{[u]} = \overline{U'}(s)f(y_3). \quad (7)$$

All the fundamental equations, initial and boundary conditions are same after the fault movement. The only adaptive boundary condition is  $\tau_{12}(y_2, y_3, t) \rightarrow 0$  as  $|y_2| \rightarrow \infty$  ( $y_3 \geq 0, t \geq 0$ ).

We solving the appear boundary value problem by revise Green's function method expanded by T. Maruyama [7],[8] and K. Rybicki [11] and correspondence principle. Let  $Q(y_1, y_2, y_3)$  be any point in the medium and  $P(\zeta_1, \zeta_2, \zeta_3)$  be any point on the fault  $F$ , then we have

$$\overline{u}(Q) = \int_F \overline{u}(P)G(P, Q), \quad (8)$$

where  $G(P, Q) = G_{12}(P, Q)d\zeta_3 - G_{13}(P, Q)d\zeta_2$  and  $G_{12}(P, Q), G_{13}(P, Q)$  are given by

$$G_{12}(P, Q) = \frac{1}{2\pi} \left[ \frac{y_2 - \zeta_2}{L^2} + \frac{y_2 - \zeta_2}{M^2} \right],$$

$$G_{13}(P, Q) = \frac{1}{2\pi} \left[ \frac{y_3 - \zeta_3}{L^2} - \frac{y_3 + \zeta_3}{M^2} \right].$$

And  $L^2 = (y_2 - \zeta_2)^2 + (y_3 - \zeta_3)^2$ ,  $M^2 = (y_2 - \zeta_2)^2 + (y_3 + \zeta_3)^2$ .

On the fault  $\zeta_2 = 0, d\zeta_2 = 0$ . From equation (7) and (8), we obtain  $\overline{u}(Q) = \frac{\overline{U'}(s)}{2\pi} \int_0^D \left[ \frac{y_2 - \zeta_2}{L^2} + \frac{y_2 - \zeta_2}{M^2} \right] f(\zeta_3) d\zeta_3$ . Taking inverse Laplace Transform with respect to time  $t_1 = t - T$ ,  $u(Q) = \frac{U'(t_1)}{2\pi} \phi(y_2, y_3) H(t_1)$ , where  $\phi(y_2, y_3) = \int_0^D \left[ \frac{y_2}{y_2^2 + (y_3 - \zeta_3)^2} + \frac{y_2}{y_2^2 + (y_3 + \zeta_3)^2} \right] f(\zeta_3) d\zeta_3$  as  $\zeta_2 = 0$  on the fault. It is to be noted that  $u = 0$  for  $t_1 = t - T \leq 0$ .

Assuming displacement, stress and strain to be zero for  $t_1 = t - T \leq 0$ , thus  $\bar{\tau}_{12} = \frac{U'(s)}{2\pi} \frac{s^\alpha}{(\frac{1}{\eta} + \frac{s^\alpha}{\mu})} \phi_1(y_2, y_3)$ . Taking inverse Laplace transform,

$$\tau_{12} = \frac{H(t_1)}{2\pi} \phi_1(y_2, y_3) L^{-1} \left\{ \frac{s^\alpha}{(\frac{1}{\eta} + \frac{s^\alpha}{\mu})} \bar{U}'(s) \right\},$$

$$\phi_1(y_2, y_3) = \frac{\partial \phi}{\partial y_2} = \int_0^D \left[ \frac{(y_3 - \zeta_3)^2 - y_2^2}{(y_2^2 + (y_3 - \zeta_3)^2)^2} + \frac{(y_3 + \zeta_3)^2 - y_2^2}{(y_2^2 + (y_3 + \zeta_3)^2)^2} \right] f(\zeta_3) d\zeta_3$$

as  $\zeta_2 = 0$  on the fault.

If we assume  $U'(t_1) = vt_1$ , where  $v$  is a constant then  $v(t_1) = \frac{\partial}{\partial t_1}(vt_1) = v$ , thus we obtain  $\tau_{12} = \frac{\eta H(t_1)V}{2\pi} \phi_1(y_2, y_3) [1 - E_\alpha(-\frac{\mu t_1^\alpha}{\eta})]$ . Similarly one can obtain

$$\tau_{13} = \frac{\eta H(t_1)V}{2\pi} \phi_2(y_2, y_3) [1 - E_\alpha(-\frac{\mu t_1^\alpha}{\eta})],$$

$$\phi_2(y_2, y_3) = \frac{\partial \phi}{\partial y_3} = -2 \int_0^D \left[ \frac{(y_3 - \zeta_3)y_2}{(y_2^2 + (y_3 - \zeta_3)^2)^2} + \frac{(y_3 + \zeta_3)y_2}{(y_2^2 + (y_3 + \zeta_3)^2)^2} \right] f(\zeta_3) d\zeta_3$$

as  $\zeta_2 = 0$  on the fault.

By the principal of superposition, the final solutions can be represented in the following forms:

$$\left. \begin{aligned} u &= u_0 + y_2 \tau_\infty(0) \left[ \frac{kt}{\mu} + \frac{t^\alpha}{\eta \Gamma(\alpha + 1)} + \frac{kt^{\alpha+1}}{\eta \Gamma(\alpha + 2)} \right] \\ &\quad + \frac{Vt_1}{2\pi} \phi(y_2, y_3) H(t_1) \\ \tau_{12} &= (\tau_{12})_0 E_\alpha(-\frac{\mu t^\alpha}{\eta}) + \tau_\infty(0) [(1 + kt) - E_\alpha(-\frac{\mu t^\alpha}{\eta})] \\ &\quad + \frac{\eta H(t_1)V}{2\pi} \phi_1(y_2, y_3) [1 - E_\alpha(-\frac{\mu t_1^\alpha}{\eta})] \\ \tau_{13} &= (\tau_{13})_0 E_\alpha(-\frac{\mu t^\alpha}{\eta}) + \frac{\eta H(t_1)V}{2\pi} \phi_2(y_2, y_3) [1 - E_\alpha(-\frac{\mu t_1^\alpha}{\eta})] \\ e_{12} &= (e_{12})_0 + \tau_\infty(0) \left[ \frac{kt}{\mu} + \frac{t^\alpha}{\eta \Gamma(\alpha + 1)} + \frac{kt^{\alpha+1}}{\eta \Gamma(\alpha + 2)} \right] \\ &\quad + \frac{Vt_1}{2\pi} H(t_1) \phi_1(y_2, y_3) \\ e_{13} &= (e_{13})_0 + \frac{Vt_1}{2\pi} H(t_1) \phi_2(y_2, y_3) \end{aligned} \right\}, \quad (9)$$

where the expressions for  $\phi$ ,  $\phi_1$ ,  $\phi_2$  are given before the equation (9) and  $E_\alpha(t)$  is Mittag-Leffler function which is defined by  $E_\alpha(t) = \sum \frac{t^p}{\Gamma(1+\alpha p)}$ ,  $p$  runs from 0 to  $\infty$  and  $0 < \alpha \leq 1$ . It has been notice that the displacement, strains and stresses are distinctive and will persist bounded overall in the model together with the upper and lower edges of the fault. The conditions for bounded stresses and strains are that the function  $f(y_3)$ ,  $f'(y_3)$  are continuous in  $0 \leq y_3 \leq D$  and either  $f''(y_3)$  is continuous in  $0 \leq y_3 \leq D$  or  $f''(y_3)$  is continuous in  $0 < y_3 < D$ , besides for a limited number of points of finite discontinuity in  $0 \leq y_3 \leq D$  or  $f''(y_3)$  is continuous in  $0 < y_3 < D$ , and remain real constants  $m < 1$  and  $n < 1$  such that  $y^m f''(y_3) \rightarrow 0$  or to a finite limit as  $y_3 \rightarrow 0^+$  and that  $(D - y_3)^n f''(y_3) \rightarrow 0$  or to a finite limit as  $y_3 \rightarrow D^-$  and  $f(D) = 0 = f'(D)$ ,  $f'(0) = 0$ .

#### 4. Numerical Results and Discussions

We assume  $f(x_3)$  to be  $f(x_3) = 1 - \frac{3x_3^2}{D^2} + \frac{2x_3^3}{D^3}$  which indulge all the conditions for aforesaid bounded strain and stresses.

Following L.M. Cathles [28], K. Aki and P.G. Richard [29], and the current research on rheological performance of crust and upper mantle by P. Chift, J. Lin, U. Barcktiausen [30], S. Karato [31], the values to the model parameters are consider as in the table below:

Parameter	Symbol used	Value taken
Rigidity	$\mu$	$3.5 \times 10^{10} N/m^2$
Viscosity	$\eta$	$3 \times 10^{19} \text{ Pa.s}$
Width of the fault F	$D$	$10 \times 10^3 \text{ meter}$
Initial stress	$(\tau_{12})_0, (\tau_{13})_0$	$20 \times 10^5 N/m^2$
Stress at infinity	$\tau_\infty(t)$	$= \tau_\infty(0)[1 + kt]$
Initial stress at infinity	$\tau_\infty(0)$	$50 \times 10^5 N/m^2$
Positive constant	$k$	$k = 10^{-9}$
Critical stress	$\tau_c$	$2 \times 10^7 N/m^2$
Parameter alpha	$\alpha$	0.4, 0.7, 1

Table 1: Different parameters and their values used in the study.

We compute the surface share strain against time for different fractional values of  $\alpha$  ( $\alpha = 0.4, \alpha = 0.7, \alpha = 1.0$ ) along the vertical fault before the commencement of the fault movement. That is we are computing  $e'_{12} = e_{12} -$



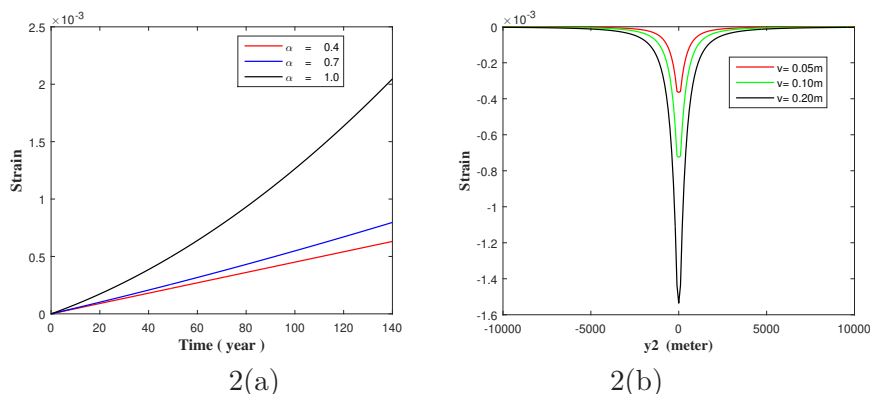


Figure 2: (a) Surface shear strain  $e'_{12}$  with different fractional values, taking  $y_3 = 0$ , before the fault movement (b) Surface shear strain  $E_{12}$  with different creep velocities, taking  $\alpha = 0.4$ ,  $y_3 = 0$ , after the fault movement

$(e_{12})_0 = \tau_\infty(0)\left[\frac{kt}{\mu} + \frac{t^\alpha}{\eta\Gamma(\alpha+1)} + \frac{kt^{\alpha+1}}{\eta\Gamma(\alpha+2)}\right]$  with time. For different fractional values of  $\alpha$ , ( $\alpha = 0.4, \alpha = 0.7, \alpha = 1.0$ ) we get various curve for the strain against time. It is observed from Fig-2(a) that under the action of  $\tau_\infty(t)$  the share strain  $e'_{12}$  increases slowly with the time. Also from this figure it is clear that for the fractional value of  $\alpha = 0.7$ , the magnitude of the strain is greater than the magnitude of the strain with fractional value of  $\alpha = 0.4$ . For  $\alpha = 1.0$  we get the magnitude of strain without fractional value and which is same as reported in the paper of P. Debnath and S. Sen [19]. We found for all the cases the magnitude of the strain before any fault movement is on the order of  $10^{-3}$  which is in conformity with the observational fact in seismically active regions during the aseismic period.

Next, we compute the surface shear strain on account of fault movement closed to the fault at the time of reinstatement of the aseismic state. Fig-2(b) shows the change of strain with respect to  $y_2$  and taking  $y_3 = 0$ . We are plotting  $E_{12} = e_{12} - (e_{12})_0 - \tau_\infty(0)\left[\frac{kt}{\mu} + \frac{t^\alpha}{\eta\Gamma(\alpha+1)} + \frac{kt^{\alpha+1}}{\eta\Gamma(\alpha+2)}\right] = \frac{Vt_1}{2\pi}H(t_1)\phi_1(y_2, y_3)$  against  $y_2$  for various creeping velocity  $v$ . It is observed that the change of shear strain release near the fault depends on the different creeping velocity  $v = 0.05\text{m/year}$ ,  $0.10\text{m/year}$ ,  $0.20\text{m/year}$ . As the surface shear strain let out close the fault on account of fault movement. So on the free surface  $y_3 = 0$ , the magnitude of this strain is negative everywhere due to release of strain. As we proceed out of the away from the fault on the surface, the magnitude of this strain release decrease rapidly. Further investigation shows that due to a creeping fault movement, the

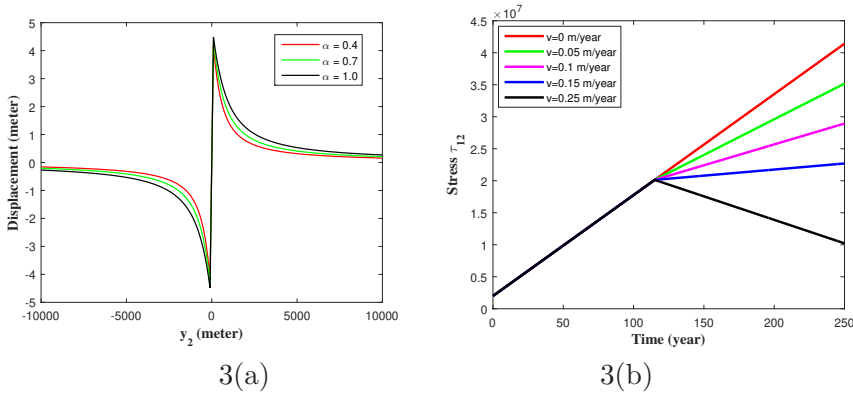


Figure 3: (a) Surface displacement along with  $y_2$  taking  $y_3 = 0$ , for the different fractional values of  $\alpha$  (b) Share stress near the mid point on the fault where  $y_2 = 0.5$  km. and  $y_3 = 5.0$  km. varying with time for different creep velocities where  $\tau_\infty$  is slowly increasing with time and fractional value of  $\alpha = 0.4$

shear strain release is on the order of  $10^{-3}$  for different creeping velocity of  $v$ . This is to be expected, as the rate of shear strain release lies between the order of  $10^{-3}$ – $10^{-7}$  in the seismically active regions, as suggested by P. Debnath and S. Sen [19].

We determine the surface displacement on account of the fault movement after reinstatement of the aseismic state  $t_1 = t - T$ , i.e.,  $u - u_0 - y_2 \tau_\infty(0) \left[ \frac{kt}{\mu} + \frac{t^\alpha}{\eta \Gamma(\alpha+1)} + \frac{kt^{\alpha+1}}{\eta \Gamma(\alpha+2)} \right] = \frac{V t_1}{2\pi} \phi(y_2, y_3) H(t_1)$  with  $y_2$ . It is observed from Fig-3(a) that this residual surface displacement depends on the fractional values of  $\alpha$ , including the different fault parameters. For different fractional values of  $\alpha$ , it has been observed some common features in displacement: (i) The residual surface displacement is attained the maximum magnitude close to the fault for both  $y_2 > 0$  and  $y_2 < 0$ . (ii) The residual surface displacement reduced quickly as we proceed out of the way from the fault on the free surface and becomes very very small for  $|y_2| \gg D$  with  $D = 10$  km. (iii) For  $y_2 > 0$  and  $y_2 < 0$ , the residual surface displacement is asymmetric and in opposite directions. In Fig-3(a), it has been observed that for  $y_2 > 0$ , the residual surface displacement attains its maximum value for  $\alpha = 1$  near the fault  $y_2 \approx 0$ . As  $y_2$  increases and  $|y_2| \rightarrow \infty$ , displacement decrease rapidly with a higher rate for smaller values of  $\alpha$  and tends to diminish as  $|y_2| \rightarrow \infty$ . We observe that in all the cases, the effect of surface displacement is very small near the fault and this effect is not

significant as  $|y_2|$  increases.

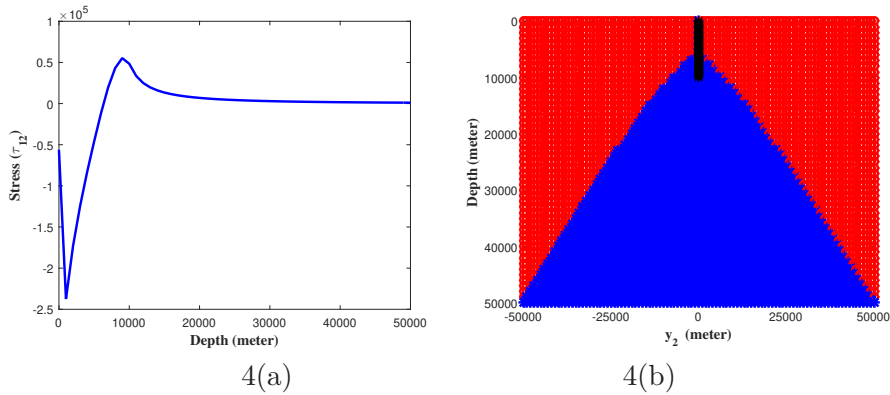


Figure 4: (a) Stress  $\tau_{12}$  due to the movement across the fault closed to the fault line (b) Stress accumulation and reduction region for the creeping movement

Next we consider the variation of share stress near the fault with time i.e. we are computing  $\tau_{12} = (\tau_{12})_0 E_\alpha(-\frac{\mu t^\alpha}{\eta}) + \tau_\infty(0)[(1 + kt) - E_\alpha(-\frac{\mu t^\alpha}{\eta})] + \frac{\eta H(t_1)V}{2\pi} \phi_2(y_2, y_3)[1 - E_\alpha(-\frac{\mu t^\alpha}{\eta})]$  with time for different creeping velocities. From the Fig-3(b), we can find the variation of share stress near the mid-point of the fault ( $y_2 = 0.5$  km, and  $y_3 = 5.0$  km) with time for different values of creep velocity, from  $v = 0$  (no creep) to  $v = 0.25$  m/year. It is found that in all cases, when fault creep was absent, there is a steady accumulation for the share stress close to the fault, with gradually decreasing rate of accumulation. If fault creep commences at  $t_1 = t - T$ , there is a reduction in the rate of accumulation of the share stress near the fault due to fault creep, and this effect is greater for larger values of the creep velocity  $v$ . For sufficiently large creep velocities, there is a gradually let out of the share stress close to the fault after  $t_1 = t - T$ , instead of accumulation of share stress and if  $v = 0.25$  m/year, there is more or less complete release of the accumulated share stress near  $F$  after a sufficient time.

In Fig-4(a), the stress  $T_{12}$  across the fault on account of the movement along the fault  $F$  where,  $T_{12} = \frac{\eta H(t_1)V}{2\pi} \phi_2(y_2, y_3)[1 - E_\alpha(-\frac{\mu t^\alpha}{\eta})]$ . We consider share stress very near to the fault line with  $y_2 = 0.5$  km the magnitude of  $T_{12}$  has been determine and  $y_3$  varies from 0 to 50 km. From the figure we observed that originally the stress is negative and its magnitude diminish up to a depth of 2 km from the top edge of the fault. Subsequently its magnitude escalate up

to the lower edge of the fault, where its be at its maximum positive value at  $y_3 = 10$  km. As we proceed downwards the accumulated stress slowly decrease and tends to zero.

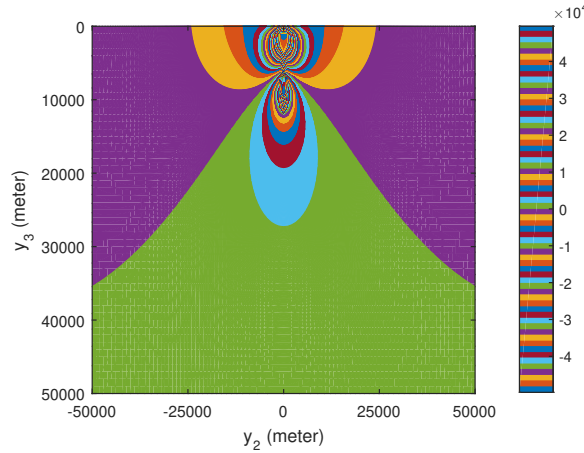


Figure 5: Contour map for stress accumulation and reduction in the medium due to the fault slip across the fault

From Fig-4(b), due to the creeping movement across the fault, we observed a clear demarcation of stress accumulation and stress reduction region. We compute here  $\tau_{12} = \frac{\eta H(t_1)V}{2\pi} \phi_2(y_2, y_3) [1 - E_\alpha(-\frac{\mu t_1^\alpha}{\eta})]$  with the depth from 0 km to 50 km and  $y_2$  from -50km to 50km, it can be notice that there is a clear separation of stress reduction region which is in red colour and stress accumulation region which is in blue colour. Now if any 2nd fault is considering in stress accumulation region the rate of accumulation of the stress near the fault will be escalate on account of the fault movement along the fault  $F$ . Accordingly the feasible movement along the 2nd fault will increase the time. Reversely if we considering a 2nd fault is situated in stress reduction region the stress rate of accumulation will be reduce on account of the fault movement along the fault  $F$ . Therefore the possible movement across the 2nd fault will delayed the time. From this view point, we have an idea about interacting faults system on relative positions. Thus our result is consistent with paper of P. Debnath and S. Sen [18] which was observed in 2015. From Fig-5, we plotted the contour map for stress release/accumulation in the medium across  $F$  due to the fault creep.

## 5. Conclusion

In this model of the lithosphere-asthenosphere system represented by fractional order Maxwell type material with long, vertical, plane strike-slip fault, our study provides overview of some physical phenomena due to the creeping strike-slip fault movement. Tectonic forces on account of mantle convection and other related occurrence are linearly increasing with time. We used Green's function technique and correspondence principle in terms of Mittag-Leffler function to determine the analytical expressions of displacement, stresses and strains for both the period. Then the model is validated by numerical results which are computed by using satisfactory model parameter. The description of the displacement, stresses and strains are analysed by considering their graphical representation. The value of the model parameters are taken an important role which are the values observed for the various earthquake in the different time. The movement of fault causes stress accumulation/release near the fault which essentially depend on not only fixed dimension of the fault and creeping velocities but also on the fractional values of the parameter  $\alpha$  and the different observational point in the medium.

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