ANALYSIS OF SPATIAL NONLINEAR
SYSTEMS OF DIFFUSION TYPE WITH DELAY

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Abstract: A spatial mathematical model of biological wastewater treatment depending on the system temperature is presented. The model investigates the relationship between the characteristics of the processes occurring at the treatment plant and offers an algorithm to solve the relevant feature of the perturbation problem and describes the process itself. The graphical relationships and dependencies between the process parameters obtained in the process of work are effective for further theoretical research aimed at optimizing the parameters of the cleaning process, namely: loading time, filter size, etc. In the process of filtration, the shape of the filter plays a significant role because it affects the performance by increasing or decreasing the filtration capacity. The influence of external factors on temperature calculations on the basis of heat transfer equations and laws of thermodynamics is investigated. The effect of water temperature and oxygen concentration on the ability to absorb activated sludge was studied. The results of the distribution of the concentration of contaminants and the temperature field during the purification of the liquid are presented. The dependence of bacterial activity on water temperature has been studied, which allows to increase the accuracy of forecasting and to develop the automation of the biological wastewater treatment process.

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1. Introduction

The search for environmentally friendly and cost-effective methods of industrial and commercial wastewater treatment has been and remains extremely relevant, especially important for large cities. Constantly growing population in settlements, intensive work of food, pharmaceutical, and microbiological many other industries, extensive infrastructure, daily lead to an increase in wastewater contaminated with various organic and inorganic substances. Given the above, the need for water treatment and purification is growing. Today, the design of wastewater treatment systems is based on common worldwide experience, which is adapted as local standards and policies governing the quality of wastewater in each country. Because wastewater can differ significantly in its ability to feed and be suitable for bacterial growth, have sufficient nutrients and low toxicity, it is necessary to test the water's ability to grow bacteria in each case. The forecast of water quality in reservoirs is carried out with the use of existing mathematical models (for generally accepted standardized indicators), which allow to determine the required degree of wastewater treatment.

2. Problem Statement

Consider a process of wastewater treatment in a biofilter in the form of a curvilinear parallelepiped $G_z = ABCDA_*B_*C_*D_*$ (Fig. 1), bounded by smooth orthogonal to each other (along the edges) equipotential surfaces $ABB_*A_* = \{z : f_1(x,y,z) = 0\}$ (in particular $f_1(x,y,z) = z - z^*$) $CDC_*C_* = \{z : f_2(x,y,z) = 0\}$ (with $f_2(x,y,z) = z - z_*$), as well as flow surfaces $ADD_*A_* = \{z : f_3(x,y,z) = 0\}$, $BCC_*B_* = \{z : f_4(x,y,z) = 0\}$, $ABCD = \{z : f_5(x,y,z) = 0\}$, $A_*B_*C_*D_* = \{z : f_6(x,y,z) = 0\}$. According to the theoretical and experimental data [4] distinguish the following components: bacteria growth and death; production of activated sludge "young" bacteria; organic contamination decomposition by bacteria; the impurities transition to a biologically non-oxidizing substance; transfer of biologically non-oxidizing substance with liquid taking into account its sorption, desorption and diffusion.
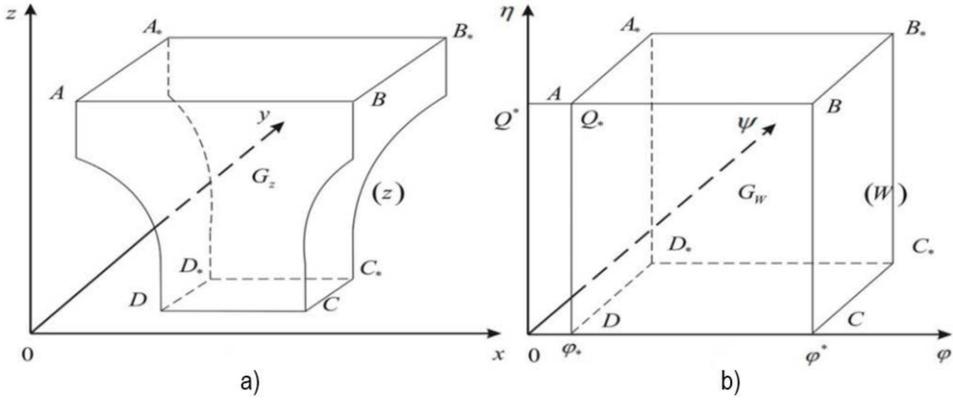
Figure 1: Physical spatial region G_z (biofilter) (a) and the corresponding integrated capacity (b)

It is assumed that: at the starting point, the biofilter is clean, its side "walls" are impermeable; at the input to the filter known (set) time distributions of all.

Considering the above statement and similar to was done in [1], [16], we can pose a problem that describes a complex change in the bacteria concentration, pollution, oxygen and temperature in the environment $G = G_z \times (0, \infty)$:

$$
\begin{cases}
\frac{\partial \bar{C}_i}{\partial t} = \vec{v} \nabla \bar{C}_i - \theta_i \bar{C}_i \bar{B} \bar{T} + \varepsilon \nabla \left[b_C \left(\bar{C}_i \left(M, t - \tau \right) \right) \nabla \bar{C}_i \right], \\
\frac{\partial \bar{B}}{\partial t} = \vec{v} \nabla \bar{B} + \theta \bar{B} \bar{K} \bar{T} K_B + w_B \\
\quad + \varepsilon \nabla \left[b_B \left(\bar{B} \left(M, t - \tau \right) \right) \nabla \bar{B} \right], \\
\frac{\partial \bar{K}}{\partial t} = \varepsilon \nabla \left[b_K \left(\bar{K} \left(M, t - \tau \right) \right) \nabla \bar{K} \right] - \vec{v} \nabla \bar{K} \\
\quad + K_K \cdot \left(K_H - \bar{K} \right) - \gamma(\bar{T}), \\
\frac{\partial \bar{T}}{\partial t} + \vec{v} \nabla \bar{T} = \varepsilon \nabla \left[b_T \left(\bar{T} \left(x, t - \tau \right) \right) \nabla \bar{T} \right] + F_T + \frac{1}{\rho \cdot c_w} \frac{\partial J}{\partial x},
\end{cases}
\tag{1}
$$

$$\bar{C}_i \mid_{ABB_*A_*} = \bar{C}_{*i}(M,t), \ \frac{\partial \bar{C}_i}{\partial \vec{n}} \mid_{CDD_*C_*} = 0,$$

$$\frac{\partial \bar{C}_i}{\partial \vec{n}} \mid_{ADD_*A_* \cup BCC_*B_* \cup ABCD \cup A_*B_*C_*D_*} = 0,$$

$$\bar{B} \mid_{ABB_*A_*} = \bar{B}_*(M,t), \ \frac{\partial \bar{B}}{\partial \vec{n}} \mid_{CDD_*C_*} = 0,$$

$$\frac{\partial \bar{B}}{\partial \vec{n}} \mid_{ADD_*A_* \cup BCC_*B_* \cup ABCD \cup A_*B_*C_*D_*} = 0,$$

$$\bar{K} \mid_{ABB_*A_*} = \bar{K}_*(M,t), \ \frac{\partial \bar{K}}{\partial \vec{n}} \mid_{CDD_*C_*} = 0,$$

$$\frac{\partial \bar{K}}{\partial \vec{n}} \mid_{ADD_*A_* \cup BCC_*B_* \cup ABCD \cup A_*B_*C_*D_*} = 0,$$

$$\bar{T} \mid_{ABB_*A_*} = \bar{T}_*(M,t), \ \frac{\partial \bar{T}}{\partial \vec{n}} \mid_{CDD_*C_*} = 0,$$

$$\frac{\partial \bar{T}}{\partial \vec{n}} \mid_{ADD_*A_* \cup BCC_*B_* \cup ABCD \cup A_*B_*C_*D_*} = 0,$$

$$(2)$$

$$\bar{C}_i(M,0) = \bar{C}_i^*(M,\tilde{t}), \ \bar{B}(M,0) = \bar{B}^*(M,\tilde{t})$$
$$\bar{K}(M,0) = \bar{K}^*(M,\tilde{t}), \ \bar{T}(M,0) = \bar{T}^*(M,\tilde{t}), \ -\tau \leq \tilde{t} \leq 0, \tag{3}$$

$$\vec{v} = \kappa \cdot grad \ \varphi, div \ \vec{v} = 0 \tag{4}$$

$$\varphi \mid_{ABB_*A_*} = \varphi_*, \varphi \mid_{CDD_{**}} = \varphi^*,$$
$$\frac{\partial \varphi}{\partial \vec{n}} \mid_{ADD_*A_* \cup BCC_*B_* \cup ABCD \cup A_*B_*C_*D_*} = 0, \tag{5}$$

where $\bar{C}_i$, mg/l – concentration of the i-th pollution in water, $\bar{T}$ K – aeration tank temperature, $\bar{B}$, mg/l – concentration of activated sludge, L_B – absorption coefficient bacteria and oxygen, w_B, $mg/l \cdot hour$ – rate of activated sludge accumulation according to the adequacy of the model, $\bar{K}$, mg/l - oxygen concentration required to maintain the best bacteria absorption by pollution, K_K – oxygen mass transfer coefficient, K_0, mg/l - water saturation concentration oxygen at a given temperature and pressure, wL is the rate of absorption of the oxygen substrate, F_T – heat transfer function, $D_C = \varepsilon b C$, $D_B = \varepsilon b K$, $D_K = \varepsilon b K$, $D_T = \varepsilon b \bar{T}$ – diffusion coefficients, $\vec{v}$ – average flow velocity, $\bar{C}_{*i}(M,t)$, $\bar{B}_*(M,t)$, $\bar{K}_*(M,t)$, $\bar{T}_*(M,t)$, $\bar{C}_i^*(M,\tilde{t})$, $\bar{B}^*(M,\tilde{t})$, $\bar{K}^*(M,\tilde{t})$, $\bar{T}^*(M,\tilde{t})$ – sufficiently smooth functions, consistent with each other on the area edges, G, M – an arbitrary point of the corresponding surface; $\varphi = \varphi(x,y,z)$ – filtration potential $0 < \varphi_* \leq \varphi \leq \varphi^* < \infty$, κ – medium coefficient of filtration, $\vec{n}$ – external normal to the corresponding surface. Here, the equality of zero normal derivatives of the desired functions along the four flow surfaces are the traditional impermeability conditions, and along the exit section CDD_*C_* – the "rapid diversion" conditions (Verigin's conditions), ε is a small parameter, τ is the delay time

$(\tau > 0)$, for $t = -\tau$ and $t = 0$ satisfy the smoothness conditions of this problem solution at $t = \tau \cdot n$ $(n = 1, 2, ...)$.

3. Literature Review

In recent years, there have been a large scientific researches number on the biochemical wastewater treatment modeling [10] describes wastewater treatment processes dynamic models that allow us to describe the treatment plants behavior under the highly dynamic conditions as well as new reactor systems for which partial differential equation descriptions are necessary to account for their distributed parameter nature (e.g. fixed bed reactors, settlers). These researches have significantly expanded the water treatment and purification understanding, heat-mass transfer, the variable parameters influence required for source information automatic control. In particular, [11] established a mathematical model for simulating a biological nutrient removal process for treating wastewater. Using phylogenetic analysis, the microbial communities in a cyclic activated sludge were quantitatively characterized, and the incorporating necessity the microbial interaction mechanism in the model was validated. Biological treatment has lower chemical and energy needs compared to chemical treatment methods [12], making it the most environmentally and economically friendly method for wastewater treatment. In particular, [14] shows that this method is effectively used as a second step in a two-process pollutant removal system from sanitary landfill leachate.

Thus, in some models, wastewater treatment is considered as a technological process with mechanical structure details without taking into consideration the dynamics in the changes of the effective and efficient operation of the filter. In others models the activated sludge and impurities relationship without taking into consideration the interacting parameters system which is clearly expressed in experiments and play an important role [8]. For example, [9] describes a sludge reduction in the anaerobic side-stream reactor experimental researches number, but the mathematical determine solids retention time does not take into account a factors number such as the temperature impact, oxygen concentration, etc. New methodologies are being developed to overcome the limitations in the practical use of wastewater treatment dynamic modelling [15].

The above-mentioned works do not take into account the impact of ambient temperature, which is one of the main factors influencing the reactions. Also, some wastewater treatment process important components have been ignored.

In particular, the longitudinal diffusion phenomenon is neglected. There are somewhat contradictory views on the expediency of taking this phenomenon into account in the literature. Thus, in the adsorption laboratory of Moscow University of Chemical Technology, it was found that at the asymptotic stage, in a wide range of flow velocities, the longitudinal diffusion erosion effect is very small compared to the mass transfer erosion effect processes. On the other hand, [13] has shown that during some substances longitudinal diffusion sorption makes certain changes in the dynamics of the process. Based on the above, it is necessary to take into account the effect of appropriate concentrations on the diffusion coefficients. It is advisable to take into account the various interactions of the process and the environment characteristics by introducing coefficients into the appropriate equations. By doing this, it is possible to analyze the processes taking place in the biofilter as a set of interinfluence and interrelated process parameters.

The purpose of this work is to develop a mathematical model of wastewater treatment from nutrients, taking into account the bacteria interaction, organic and biologically non-oxidizing substances influenced by the temperature. This will allow to more accurately predict the behavior and wastewater treatment process performance, and further enables automated control of impurities deposition in the biological filters depending on the initial data of the aquatic environment.

4. Materials and Methods

Assume that the filtration problem stated in (4)-(5) on the spatial conformal mapping $G_w \mapsto G_z$ where

$$G_w = \{w = (\varphi, \psi, \eta): \; \varphi_* < \varphi < \varphi^*, \, 0 < \psi < Q_*, \, 0 < \eta < Q^*\}$$

is the corresponding region of the complex potential G_z; $\psi = \psi(x,y,z)$, $\eta = \eta(x,y,z)$ – flow functions are complexly conjugate with $\varphi = \varphi(x,y,z)$, namely, such that: $grad\varphi = grad\psi \times grad\eta$ is solved by [1] in particular, a dynamic grid and a velocity field $\vec{v}$ are constructed, the filtration flow $Q = Q_* \cdot Q^*$ is calculated. Then, having replaced the variables $x = x(\varphi, \psi, \eta)$, $y = y(\varphi, \psi, \eta)$, $z = z(\varphi, \psi, \eta)$ in the system (1) and conditions (2)-(3) one can arrive at the corresponding convective-diffusion-mass transfer problem for the $G_\omega = G_w \times (0, \infty)$ domain, the solution of which is delayed by τ at time intervals $[(n-1)\tau, n\tau]$, $n = 1, 2, ...$ let us replace by the successive solution of n problems

without delay:

$$
\begin{cases}
\frac{\partial C_i^{[n]}}{\partial t} = \vec{v}\nabla C_i^{[n]} - \theta_i C_i^{[n]} B^{[n]} T^{[n]} \\
+ \varepsilon \nabla \left[b_C \left(C_i^{[n]}(\varphi, \psi, \eta, t - \tau) \right) \nabla C_i^{[n]} \right], \\
\frac{\partial B^{[n]}}{\partial t} = \vec{v}\nabla B^{[n]} + \theta B^{[n]} K^{[n]} T^{[n]} K_B + w_B \\
+ \varepsilon \nabla \left[b_B \left(B^{[n]}(\varphi, \psi, \eta, t - \tau) \right) \nabla B^{[n]} \right], \\
\frac{\partial K^{[n]}}{\partial t} = \varepsilon \nabla \left[b_K \left(K^{[n]}(\varphi, \psi, \eta, t - \tau) \right) \nabla K^{[n]} \right] - \vec{v}\nabla K^{[n]} \\
+ K_K \left(B^{[n]} \right) \cdot \left(K_H - K^{[n]} \right) - \gamma(T^{[n]}), \\
\frac{\partial T^{[n]}}{\partial t} + \vec{v}\nabla T^{[n]} = \varepsilon \nabla \left[b_T \left(T^{[n]}(\varphi, \psi, \eta, t - \tau) \right) \nabla T^{[n]} \right] \\
+ F_T + \frac{1}{\rho \cdot c_w} \frac{\partial J}{\partial x},
\end{cases}
\tag{6}
$$

$$
C_i^{[n]}(\varphi_*, \psi, \eta, t) = C_{*i}(\psi, \eta, t),
$$

$$
C_i^{[n]}(\varphi, \psi, \eta, t - \tau) = C_i^{[n-1]}(\varphi, \psi, \eta, t - \tau),
$$

$$
b_{Cn\tau}(\varphi, \psi, \eta, t) = b_C \left(C_i^{[n]}(\varphi, \psi, \eta, t - \tau) \right)
$$

$$
= b_C \left(C_i^{[n-1]}(\varphi, \psi, \eta, t - \tau) \right),
$$

$$
C_i^{[0]}(\varphi, \psi, \eta, 0) = C_i^*(\varphi, \psi, \eta, 0),
$$

$$
B^{[n]}(\varphi_*, \psi, \eta, t) = B_*(\psi, \eta, t),
$$

$$
B^{[n]}(\varphi, \psi, \eta, t - \tau) = B^{[n]}(\varphi, \psi, \eta, t - \tau),
$$

$$
b_{Bn\tau}(\varphi, \psi, \eta, t) = b_B \left(B^{[n]}(\varphi, \psi, \eta, t - \tau) \right)
$$

$$
= b_B \left(B^{[n-1]}(\varphi, \psi, \eta, t - \tau) \right),
$$

$$
B^{[0]}(\varphi, \psi, \eta, 0) = B^*(\varphi, \psi, \eta, 0),
$$

$$
K^{[n]}(\varphi_*, \psi, \eta, t) = K_*(\psi, \eta, t),
$$

$$
K^{[n]}(\varphi, \psi, \eta, t - \tau) = K^{[n]}(\varphi, \psi, \eta, t - \tau),
$$

$$
b_{Kn\tau}(\varphi, \psi, \eta, t) = b_K \left(K^{[n]}(\varphi, \psi, \eta, t - \tau) \right)
$$

$$
= b_K \left(K^{[n-1]}(\varphi, \psi, \eta, t - \tau) \right),
$$

$$
K^{[0]}(\varphi, \psi, \eta, 0) = K^*(\varphi, \psi, \eta, 0),
$$

$$
T^{[n]}(\varphi_*, \psi, \eta, t) = T_*(\psi, \eta, t),
$$

$$T^{[n]}\left(\varphi,\psi,\eta,t-\tau\right)=T^{[n]}\left(\varphi,\psi,\eta,t-\tau\right),$$

$$b_{Tn\tau}\left(\varphi,\psi,\eta,t\right)=b_{T}\left(T^{[n]}\left(\varphi,\psi,\eta,t-\tau\right)\right)$$

$$=b_{T}\left(T^{[n-1]}\left(\varphi,\psi,\eta,t-\tau\right)\right),$$

$$T^{[0]}\left(\varphi,\psi,\eta,0\right)=T^{*}\left(\varphi,\psi,\eta,0\right),$$

$$\begin{aligned}
C_{i\psi}(\varphi,0,\eta,t)&=C_{i\psi}(\varphi,Q_*,\eta,t)\\
=C_{i\eta}(\varphi,\psi,0,t)&=C_{i\eta}(\varphi,\psi,Q^*,t)=0,\\
B_{\psi}(\varphi,0,\eta,t)&=B_{\psi}(\varphi,Q_*,\eta,t)\\
=B_{\eta}(\varphi,\psi,0,t)&=B_{\eta}(\varphi,\psi,Q^*,t)=0,\\
K_{\psi}(\varphi,0,\eta,t)&=K_{\psi}(\varphi,Q_*,\eta,t)\\
=K_{\eta}(\varphi,\psi,0,t)&=K_{\eta}(\varphi,\psi,Q^*,t)=0,\\
T_{\psi}(\varphi,0,\eta,t)&=T_{\psi}(\varphi,Q_*,\eta,t)\\
=T_{\eta}(\varphi,\psi,0,t)&=T_{\eta}(\varphi,\psi,Q^*,t)=0.
\end{aligned} \tag{7}$$

The solution of equations (6)-(7) with $O(\varepsilon^2)$ accuracy in the strong consistency assumption of initial and boundary conditions along the edges and at the angular points of the G_ω region (see, for example, [1]) is sought in the following asymptotic series form:

$$C_i^{[n]} = C_{i,0}^{[n]} + \sum_{j=1}^{m}\varepsilon^j C_{i,j}^{[n]} + \sum_{j=0}^{m}\varepsilon^j \Pi_{i,j}^{[n]}$$
$$+\sum_{j=0}^{m}\varepsilon^j \overset{\smile}{\Pi}_{i,j}^{[n]} + \sum_{j=0}^{m+1}\varepsilon^{j/2}\tilde{\Pi}_{i,j}^{[n]} + \sum_{j=0}^{m+1}\varepsilon^{j/2}\tilde{\tilde{\Pi}}_{i,j}^{[n]}$$
$$+\sum_{j=0}^{m+1}\varepsilon^{j/2}\widehat{\Pi}_{i,j}^{[n]} + \sum_{j=0}^{m+1}\varepsilon^{j/2}\widehat{\widehat{\Pi}}_{i,j}^{[n]} + R_C^{[n]},$$
$$B^{[n]} = B_0^{[n]} + \sum_{j=1}^{m}\varepsilon^j B_j^{[n]} + \sum_{j=0}^{m}\varepsilon^j N_j^{[n]}$$
$$+\sum_{j=0}^{m}\varepsilon^j \overset{\smile}{N}_j^{[n]} + \sum_{j=0}^{m+1}\varepsilon^{j/2}\tilde{N}_j^{[n]} + \sum_{j=0}^{m+1}\varepsilon^{j/2}\tilde{\tilde{N}}_j^{[n]}$$
$$+\sum_{j=0}^{m+1}\varepsilon^{j/2}\widehat{N}_j^{[n]} + \sum_{j=0}^{m+1}\varepsilon^{j/2}\widehat{\widehat{N}}_j^{[n]} + R_T,$$

$$K^{[n]} = K_0^{[n]} + \sum_{j=1}^{m} \varepsilon^j K_j^{[n]} + \sum_{j=0}^{m} \varepsilon^j M_j^{[n]}$$

$$+ \sum_{j=0}^{m} \varepsilon^j \breve{M}_j^{[n]} + \sum_{j=0}^{m+1} \varepsilon^{j/2} \tilde{M}_j^{[n]} + \sum_{j=0}^{m+1} \varepsilon^{j/2} \tilde{\tilde{M}}_j^{[n]}$$

$$+ \sum_{j=0}^{m+1} \varepsilon^{j/2} \widehat{M}_j^{[n]} + \sum_{j=0}^{m+1} \varepsilon^{j/2} \widehat{\widehat{M}}_j^{[n]} + R_T,$$

$$T^{[n]} = T_0^{[n]} + \sum_{j=1}^{m} \varepsilon^j T_j^{[n]} + \sum_{j=0}^{m} \varepsilon^j P_j^{[n]}$$

$$+ \sum_{j=0}^{m} \varepsilon^j \breve{P}_j^{[n]} + \sum_{j=0}^{m+1} \varepsilon^{j/2} \tilde{P}_j^{[n]} + \sum_{j=0}^{m+1} \varepsilon^{j/2} \tilde{\tilde{P}}_j^{[n]}$$

$$+ \sum_{j=0}^{m+1} \varepsilon^{j/2} \widehat{P}_j^{[n]} + \sum_{j=0}^{m+1} \varepsilon^{j/2} \widehat{\widehat{P}}_j^{[n]} + R_T,$$

(8)

where $R_C^{[n]}$, $R_B^{[n]}$, $R_K^{[n]}$, $R_T^{[n]}$, residual members ($i = 1, 2$), $C_{i,j}^{[n]}$, $B_j^{[n]}$, $K_j^{[n]}$, $T_j^{[n]}$, $j = \overline{0, \ m}$, – members of the regular parts of the asymptote $\Pi_{i,j}^{[n]}(\xi,\psi,\eta,t)$, $N_j^{[n]}(\xi,\psi,\eta,t)$, $M_j^{[n]}(\xi,\psi,\eta,t)$, $P_j^{[n]}(\xi,\psi,\eta,t)$, – functions of the boundary layer type in the neighborhood $\varphi = \varphi^*$, (corrections at the exit of the filter) ($j = \overline{0,1}$), $\breve{\Pi}_{i,j}^{[n]}(\breve{\xi}, \psi, \eta, t)$, $\breve{N}_j^{[n]}(\breve{\xi}, \psi, \eta, t)$ $\breve{M}_j^{[n]}(\breve{\xi}, \psi, \eta, t)$, $\breve{P}_j^{[n]}(\breve{\xi}, \psi, \eta, t)$ in the vicinity $\varphi = \varphi_*$ (corrections at the entrance to the filter) ($i = \overline{0, 1}$), and functions

$\tilde{\Pi}_{i,j}\left(\varphi, \tilde{\psi}, \eta, t\right)$, $\tilde{\tilde{\Pi}}_{i,j}\left(\varphi, \tilde{\tilde{\psi}}, \eta, t\right)$, $\widehat{\Pi}_{i,j}(\varphi, \psi, \tilde{\eta}, t)$, $\widehat{\widehat{\Pi}}_{i,j}\left(\varphi, \psi, \tilde{\tilde{\eta}}, t\right)$,

$\tilde{N}_i\left(\varphi, \tilde{\psi}, \eta, t\right)$, $\tilde{\tilde{N}}_i\left(\varphi, \tilde{\tilde{\psi}}, \eta, t\right)$, $\widehat{N}_i(\varphi, \psi, \tilde{\eta}, t)$, $\widehat{\widehat{N}}_i\left(\varphi, \psi, \tilde{\tilde{\eta}}, t\right)$,

$\tilde{M}_i\left(\varphi, \tilde{\psi}, \eta, t\right)$, $\tilde{\tilde{M}}_i\left(\varphi, \tilde{\tilde{\psi}}, \eta, t\right)$, $\widehat{M}_i(\varphi, \psi, \tilde{\eta}, t)$, $\widehat{\widehat{M}}_i\left(\varphi, \psi, \tilde{\tilde{\eta}}, t\right)$ and $\tilde{P}_i\left(\varphi, \tilde{\psi}, \eta, t\right)$,

$\tilde{\tilde{P}}_i\left(\varphi, \tilde{\tilde{\psi}}, \eta, t\right)$, $\widehat{P}_i(\varphi, \psi, \tilde{\eta}, t)$, $\widehat{\widehat{P}}_i\left(\varphi, \psi, \tilde{\tilde{\eta}}, t\right)$ ($i = \overline{0.3}$) in the rounds $\psi = 0$, $\psi = Q_*$, $\eta = 0$, $\eta = Q^*$ (corrections on the filter sidewalls), respectively; $\xi = \frac{\varphi^* - \varphi}{\varepsilon}$, $\breve{\xi} = \frac{\varphi - \varphi_*}{\varepsilon}$, $\tilde{\psi} = \frac{\psi}{\sqrt{\varepsilon}}$, $\tilde{\tilde{\psi}} = \frac{Q_* - \psi}{\sqrt{\varepsilon}}$, $\tilde{\eta} = \frac{\eta}{\sqrt{\varepsilon}}$, $\tilde{\tilde{\eta}} = \frac{Q^* - \eta}{\sqrt{\varepsilon}}$ – "stretches" of the corresponding variables.

The substitution of eq. (8) into eq. (6)-(7) and the performance of standard procedure of coefficients "equalization" at identical ϵ degrees, provide the

following results $C_{i,j}^{[n]}(x,t)$, $B_j^{[n]}(x,t)$, $K_j^{[n]}(x,t)$, $T_j^{[n]}(x,t)$, $\left(j = \overline{0,m}\right)$:

$$
\begin{cases}
\frac{\partial C_{i,0}^{[n]}}{\partial t} - v\frac{\partial C_{i,0}^{[n]}}{\partial x} + \theta C_{i,0}^{[n]} B_0^{[n]} T_0^{[n]} = 0; \\
\frac{\partial B_0^{[n]}}{\partial t} - v\frac{\partial B_0^{[n]}}{\partial x} - B_0^{[n]} K_0^{[n]} T_0^{[n]} K_B - w_B = 0, \\
\frac{\partial K_0^{[n]}}{\partial t} + v\frac{\partial K_0^{[n]}}{\partial x} - K_K(B) \cdot \left(K_H - K_0^{[n]}\right) - \gamma(T_0^{[n]}) = 0, \\
\frac{\partial T_0^{[n]}}{\partial t} + v\frac{\partial T_0^{[n]}}{\partial x} - F_T + \frac{1}{\rho \cdot c_w}\frac{\partial J}{\partial x} = 0, \\
C_i^{[n]}(\varphi_*, \psi, \eta, t) = C_{*i}(\psi, \eta, t), \\
C_i^{[n]}(\varphi, \psi, \eta, t - \tau) = C_i^{[n-1]}(\varphi, \psi, \eta, t - \tau), \\
B^{[n]}(\varphi_*, \psi, \eta, t) = B_*(\psi, \eta, t), \\
B^{[n]}(\varphi, \psi, \eta, t - \tau) = B^{[n]}(\varphi, \psi, \eta, t - \tau), \\
K^{[n]}(\varphi_*, \psi, \eta, t) = K_*(\psi, \eta, t), \\
K^{[n]}(\varphi, \psi, \eta, t - \tau) = K^{[n]}(\varphi, \psi, \eta, t - \tau), \\
T^{[n]}(\varphi_*, \psi, \eta, t) = T_*(\psi, \eta, t), \\
T^{[n]}(\varphi, \psi, \eta, t - \tau) = T^{[n]}(\varphi, \psi, \eta, t - \tau),
\end{cases}
\tag{9}
$$

$$
\begin{cases}
\frac{\partial C_{i,j}^{[n]}}{\partial t} - v\nabla C_j^{[n]} + \theta C_{i,j}^{[n]} B_j^{[n]} T_j^{[n]} = \tilde{C}_j^{[n]}; \\
\frac{\partial B_j^{[n]}}{\partial t} - v\nabla B_j^{[n]} - B_j^{[n]} K_j^{[n]} T_j^{[n]} K_B = \tilde{B}_j^{[n]}, \\
\frac{\partial K_j^{[n]}}{\partial t} + v\nabla K_j^{[n]} + K_K(B) K_j^{[n]} - \gamma(T_j^{[n]}) = \tilde{K}_j^{[n]}, \\
\frac{\partial T_j^{[n]}}{\partial t} + v\nabla T_j^{[n]} = \tilde{T}_j^{[n]}, \\
C_i^{[n]}\big|_{x=0} = 0, \quad B^{[n]}\big|_{x=0} = 0, \\
K^{[n]}\big|_{x=0} = 0, \quad K^{[n]}\big|_{x=0} = 0, \\
C_i^{[n]}\big|_{t=0} = 0, \quad B^{[n]}\big|_{t=0} = 0, \\
K^{[n]}\big|_{t=0} = 0, \quad K^{[n]}\big|_{t=0} = 0,
\end{cases}
\tag{10}
$$

where:

$$
\tilde{C}_j^{[n]} = b_{Cn\tau}\Delta C_{i,j-1}^{[n]} + \nabla\left(C_{i,j}^{[n]} b_{Cn\tau}\right),
$$

$$
\tilde{B}_j^{[n]} = b_{Bn\tau}\Delta C_{i,j-1}^{[n]} B_{j-1}^{[n]} + \nabla\left(B_j^{[n]} b_{Bn\tau}\right),
$$

$$
\tilde{K}_j^{[n]} = b_{Kn\tau}\Delta K_{j-1}^{[n]} + \nabla\left(K_{j-1}^{[n]} b_{Kn\tau}\right), \quad \tilde{T}_j^{[n]} = b_{Tn\tau}\Delta T_{j-1}^{[n]} + \nabla\left(T_{j-1}^{[n]} b_{Tn\tau}\right).
$$

Subtasks for finding functions $\Pi_{i,j}^{[n]}(\xi, \psi, \eta, t)$, $N_j^{[n]}(\xi, \psi, \eta, t)$, $M_j^{[n]}(\xi, \psi, \eta, t)$, $P_j^{[n]}(\xi, \psi, \eta, t)$, $\breve{\Pi}_{i,j}^{[n]}(\breve{\xi}, \psi, \eta, t)$, $\breve{N}_j^{[n]}(\breve{\xi}, \psi, \eta, t)$ $\breve{M}_j^{[n]}(\breve{\xi}, \psi, \eta, t)$, $\breve{P}_j^{[n]}(\breve{\xi}, \psi, \eta, t)$,

$$\tilde{\Pi}_{i,j}\left(\varphi,\tilde{\psi},\eta,t\right), \tilde{\tilde{\Pi}}_{i,j}\left(\varphi,\tilde{\tilde{\psi}},\eta,t\right), \widehat{\Pi}_{i,j}\left(\varphi,\psi,\tilde{\eta},t\right), \widehat{\widehat{\Pi}}_{i,j}\left(\varphi,\psi,\tilde{\tilde{\eta}},t\right),$$

$$\tilde{N}_i\left(\varphi,\tilde{\psi},\eta,t\right), \tilde{\tilde{N}}_i\left(\varphi,\tilde{\tilde{\psi}},\eta,t\right), \widehat{N}_i\left(\varphi,\psi,\tilde{\eta},t\right), \widehat{\widehat{N}}_i\left(\varphi,\psi,\tilde{\tilde{\eta}},t\right),$$

$$\tilde{M}_i\left(\varphi,\tilde{\psi},\eta,t\right), \tilde{\tilde{M}}_i\left(\varphi,\tilde{\tilde{\psi}},\eta,t\right), \widehat{M}_i\left(\varphi,\psi,\tilde{\eta},t\right), \widehat{\widehat{M}}_i\left(\varphi,\psi,\tilde{\tilde{\eta}},t\right) \text{ and } \tilde{P}_i\left(\varphi,\tilde{\psi},\eta,t\right),$$

$$\tilde{\tilde{P}}_i\left(\varphi,\tilde{\tilde{\psi}},\eta,t\right), \widehat{P}_i\left(\varphi,\psi,\tilde{\eta},t\right), \widehat{\widehat{P}}_i\left(\varphi,\psi,\tilde{\tilde{\eta}},t\right) \text{ [1, 6, 7, 2, 5, 3].}$$

5. Results

The following data were used for computational experiments: $Q = 7.2\ m^3/hour$, $V = 1000 m^3$, $k_i = 10$, $w_B = 0.0736$, $v_1 = 0.26\ m/hour$, $v_B = 0.092\ m/hour$, $v_K = 0.053\ m/hour$, $v_T = 0.53\ m/hour$, $D_t = D_{C_i} = 1$, $D_B = D_K = 0.8$, $\theta_1 = 10^{-4}$, $\theta_2 = 10^{-3}$, $\theta_3 = 1.2 \cdot 10^{-3}$, $\theta_4 = 0.8 \cdot 10^{-3}$, $\theta_5 = 1.1 \cdot 10^{-4}$, $\theta_6 = 1.6 \cdot 10^{-4}$, $\theta_7 = 1.4 \cdot 10^{-5}$, $\theta_8 = 1.3 \cdot 10^{-5}$, $\theta_{11} = 1.5 \cdot 10^{-4}$, $c_v = 4.2\ J/kg \cdot K$, $\rho_v = 1000\ kg/m^3$, $l = 100\ m$, $h = 2\ m$, $b = 5\ m$, $t_{vx} = 291\ °K$, $\beta = 0.025$, $\lambda_p = 0.02 \cdot 10^6 W/m \cdot K$, $t_{tep} = 343\ °K$, $\lambda_{st} = 46.5 W/m \cdot K$, $t_z = -0,0003 \cdot t^2 + 0.2171 \cdot t + 6.8993$, $\mu = 0.02$, $t_{zd} = 280\ °K$, $a = 135\ W/m^2 \cdot K$, $\varepsilon = 0.01$.

Experimental data presented by [8, 7] were used to confirm the proposed mathematical model adequacy. The authors carried out 5 experiments where ammoniacal and organic nitrogen concentrations were measured at the treatment plant input and at the output after a given time (Table 1). The corresponding nitrogen concentration at the structure output was calculated and presented in table I along with the data on installation and input pollution concentration.

As it can be observed from Table 1, the relative error of the obtained results for all experiments does not exceed 3 %, which indicates the adequacy of the aerobic wastewater treatment process. The computer simulation results demonstrate the functions dynamics nature: a) over the time at each point in the filter length; b) along the filter length at different time steps $t = 0$, $t = 50$, $t = 100$, $t = 150$, $t = 200\ days$; c) over the time at different filter points $x = 0$, $x = 25$, $x = 50$, $x = 75$, $x = 100\ m$. Fig. 2 – Fig. 6 show that the bacteria absorb contamination most actively at the filter beginning and absorption efficiency for each contamination concentration is different. The contaminants concentration at the filter output changes because of the water temperature augmentation. This indicates that the water influences temperature the bacterial contamination absorption efficiency.

Fig. 5 shows that the most active bacteria reproduce at the filter beginning.

Table 1: Experimental data

Indicator		Experiment #				
		1	2	3	4	5
Building Entrance Concentration	N_{gen}^{in}	5.76	11.66	15.89	30.73	41.23
	N_{org}^{in}	0.4	0.69	1.04	2.38	3.58
	$N_{NH_3}^{in}$	1.86	3.85	4.98	10	12.67
Building Exit Concentration (Experiment)	N_{gen}^{out}	4.66	9.46	13.09	25.53	34.63
	N_{org}^{out}	0.17	0.3	0.52	1.31	2.08
	$N_{NH_3}^{out}$	0.1	0.4	0.62	1.4	1.8
Building Exit Concentration (Modeling)	N_{gen}	4.66	9.46	13.09	25.52	34.63
	time, h	13.66	13.36	11.58	10.69	9.5
	N_{org}	0.17	0.30	0.52	1.3	2.07
	time, h	24.35	25.24	22.27	20.19	18.71
	N_{NH_3}	0.09	0.38	0.61	1.4	1.83
	time, h	24.05	20.79	20.49	24.35	20.49
Relative Error, %	N_{gen}	0.075	0.034	0.03	0.01	0.019
	N_{org}	2.52	0.43	1.19	0.305	0.22
	N_{NH_3}	1.4	2.77	0.25	0.7	1.72
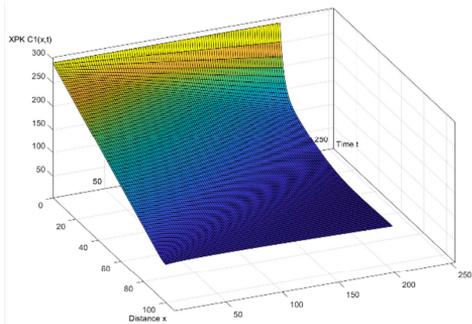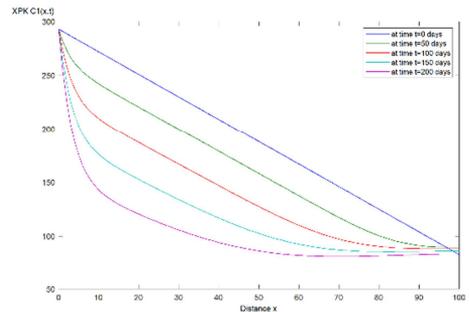
a)   b)

Figure 2: N_{gen} concentration distribution: a) over the whole filter length with a time; b) along the length of the filter at different points in time
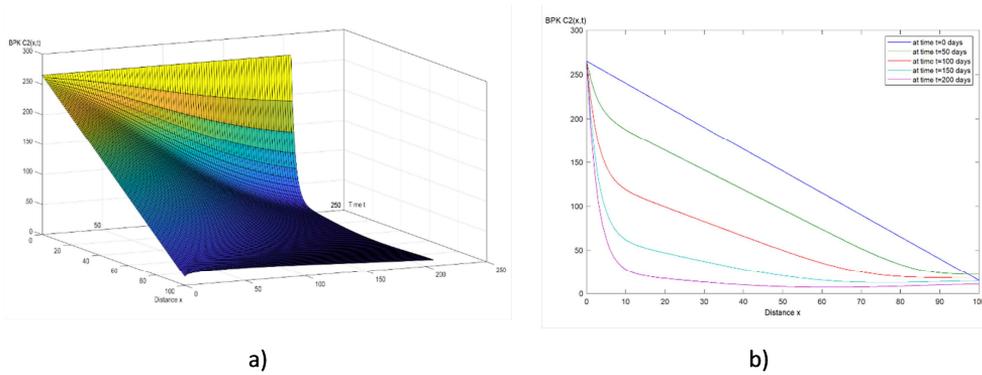
a)
b)

Figure 3: N_{org} concentration distribution: a) over the entire length of the filter over time; b) along the length of the filter at different points in a time
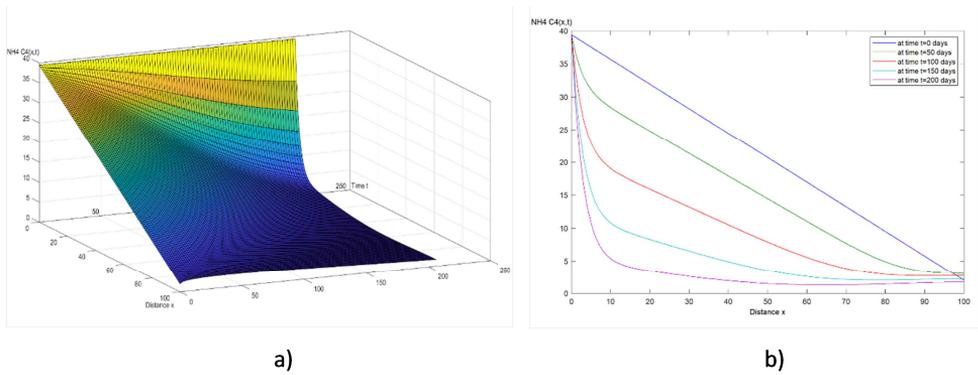

a)
b)

Figure 4: N_{org} concentration distribution: a) over the entire length of the filter over time; b) along the length of the filter at different points in a time
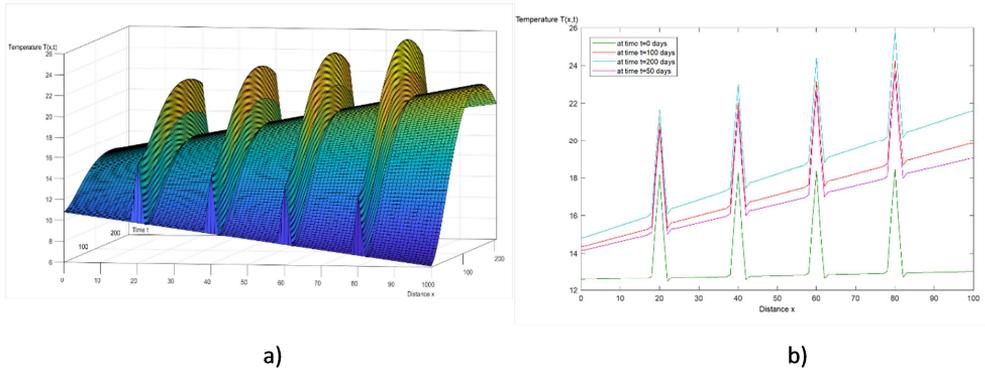
a)   b)

Figure 5: N_{NH_3} concentration distribution: a) over the entire length of the filter over time; b) along the length of the filter at different points in a time

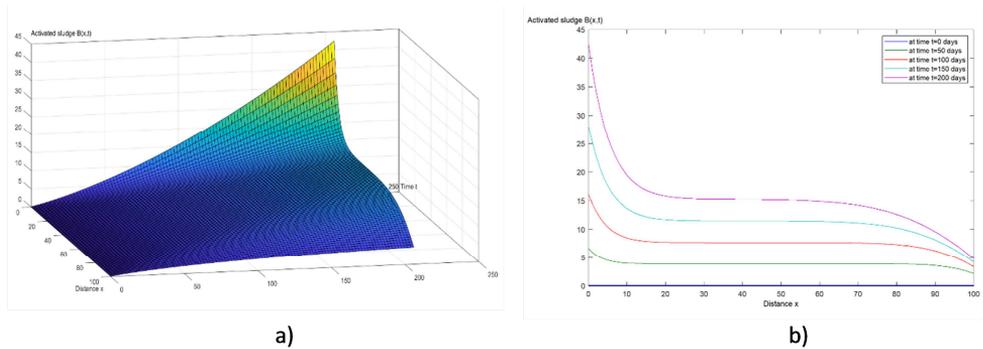a)   b)

Figure 6: Temperature field distribution: a) over the entire length of the filter over time; b) along the length of the filter at different points in a time

The presence of permanent high concentration contamination, oxygen and a permanent input water temperature provide the activated sludge development. Nevertheless, there is a trend to a more delayed reproduction at the filter exit. Numerous factors affect the water temperature change, but the heat supply by the coolant increases the water temperature only by 6 degrees, as illustrated in Fig. 5 (b). This is due to the fact the heat transfer from the heated air to the water is slow and the time during which the heated air passes from the supply point to the reactor surface is very small, as well as the small area of interaction. The ambient temperature has a much greater effect on the heating or cooling of the water since the areas of interaction on the surface and through the sidewalls of the reactor are much larger than the area of interaction with the coolant.

6. Conclusion

A biological wastewater treatment process mathematical model was derived taking into account the bacteria interaction, organic and biologically non-oxidizing substances under the diffusion conditions and mass transfer perturbation and the temperature regimes influence. A method along with an algorithm for solving the corresponding nonlinear perturbed convection-diffusion-heat-mass transfer problem was developed. The derived mathematical equations solutions were obtained using the Matlab software and his "pdepe" function. The calculations results of the pollution concentration distribution and temperature as a function of the fluid purification time are presented. The results obtained in this work make it possible to more accurately predict and automate the technological biological wastewater treatment processes.

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