APPLICATION OF CONTINUOUS WAVELET TRANSFORM IN THE ANALYSIS OF ELECTROCARDIOGRAM SIGNALS

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Abstract: The electrocardiogram is known as a primary and powerful diagnostic tool that provides all essential information about the health of our heart. The feature extraction of electrocardiogram, such as R-peak detection, is the central core of any electrocardiogram analysis. Study of electrocardiogram in wavelet domain using continuous wavelet transform with well-known wavelets and other proposed wavelets for this investigation is found to be helpful and yields reasonably reliable results. In order to validate this method, we apply it to several MIT-BIH database records. The continuous wavelet transform with one of the proposed wavelets namely, Mxr-1, achieves 99.97 % sensitivity, 99.89 % positive predictivity, and 0.135 % detection error for accurate detection of R peaks in comparison with the well-known standard wavelets such as Morlet, Mexican hat and Daubechies 4 and two other proposed mother wavelets Mxr-2 and Mxr-3.

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Table 1: Amplitude and duration of different waves in ECG

Amplitude (mV)		Duration (seconds)	
P Wave	0.25mV	PR interval	0.12s to 0.20s
R Wave	1.60mV	QT interval	0.35s to 0.44s
Q Wave	25% of R wave	ST interval	0.05s to 0.15s
T Wave	0.1 to 0.5mV	QRS	0.09s

1. Introduction

An electrocardiogram (ECG) represents the heart's electrical activity, where each heartbeat is displayed as a series of electrical waves characterized by peaks and valleys. It is used clinically in diagnosing various abnormalities and conditions associated with the heart. To interpret an ECG, it is necessary to understand the components making up the ECG signal. One typical ECG trace of the cardiac cycle (heartbeat) consists of different types of waves such as P, QRS, and T, as shown in Figure 1. Each of these waves represents a specific location in the structure of the heart. The P-wave represents the activation of the heart's upper chambers, the atria, while the QRS complex and T-wave represent the excitation of the ventricles or the lower chamber of the heart. The performance of ECG analysis depends mainly on the accurate and reliable detection of the QRS complex, T and P waves.

Any ECG signal gives two kinds of information such as, the duration of the electrical wave crossing the heart, which in turn decides whether the electrical activity is regular or slow or irregular and the amount of electrical activity passing through the heart muscle which in turn enables to find whether the parts of the heart are too large or overworked. Typically, the frequency range of an ECG signal is (0.05 - 100) Hz and its dynamic range is (1 - 10) mV. The amplitude and duration of an ECG are summarized in Table 1.

Among all the waves in an ECG, the QRS complex is the most striking waveform within the electrocardiogram (ECG). So, the detection of it is the most critical task in automatic ECG signal analysis. QRS complex reflects the electrical activity within the heart during the ventricular contraction. The time of its occurrence and its shape provides much information about the current state of the heart. Due to its characteristic shape (Figure 1), it serves as the basis for the automated determination of the heart rate, as an entry point for the cardiac cycle classification schemes, and often it is also used in ECG data compression. In that sense, QRS detection provides the fundamentals for
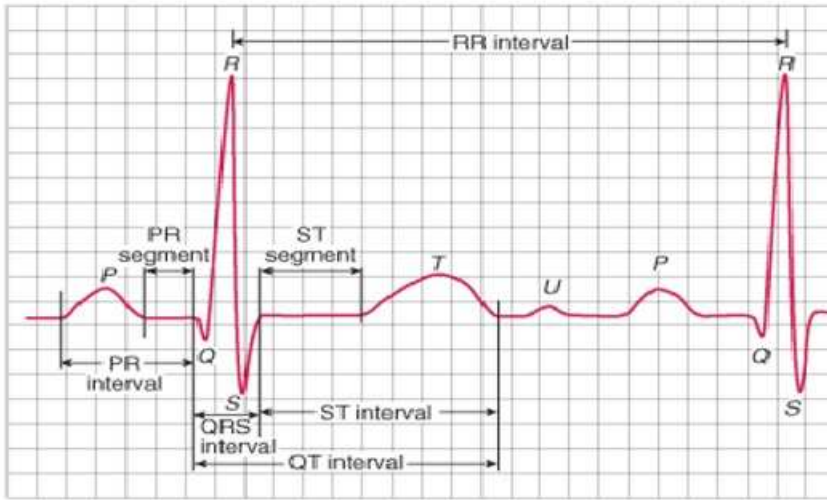
Figure 1: Sketch of an electrocardiogram (ECG) cycle

almost all automated ECG analysis. Once the QRS complex is identified, a more detailed examination of the ECG signal, including the heart rate, ST-segment etc can be realized.

Various methods are in vogue to analyze ECG signals of which Coast et al. [1], use the hidden Markov models which observes the data sequence by a probability function that varies according to the state of an underlying (hidden) Markov chain. Hu [2], uses Neural network approach employing adaptive non-linear predictors. The idea was to predict the current signal value from its past values and apply suitable filters to attenuate the noise. Poli et al. [3], use genetic algorithm with an aim to get optimal polynomial filters for preprocessing stage and parameters for the decision making stage whereas, Benitez et al. [4], proposed an algorithm for QRS detection using the first differential of the ECG signal and its Hilbert transformed signal to locate the R wave peaks in the ECG waveform. Wavelets and all their variants also are being used in ECG analysis. Martinez et al. in [5], designed and implemented a wavelet packet based algorithm for QRS detection and R/S wave identification. Kadambe et al. [6], describe a QRS complex detector based on the discrete wavelet transform (DWT) and continuous wavelet transform by Aqil et al. in [7]. This study implements a method based on continuous wavelet transform with different mother wavelets for locating R peaks. The performance parameters (sensitivity, positive predictivity and detection error) in detecting R peaks are found to agree with well-known wavelets.

Continuous Wavelet Transform

A Wavelet is a small wave with energy concentrated in a narrow time interval for analyzing transient, non-stationary or time-varying signals [8]. There are various Wavelets available to be used in a large variety of applications. Wavelet families include Biorthogonal, Haar, Coiflet, Symlet, Daubechies Wavelets, etc. Some features which make them useful are:

- Wavelets are localized in both time and frequency.

- For analyzing non-stationary signals such as ECG, which have frequent level variations and uneven features.

- Wavelet separates a signal into multiresolution components

Wavelet transform is a time-scale representation of a signal that has been used effectively in a variety of applications. It is a linear process that decomposes a given signal into several scales associated with different frequency components and analyzes each scale with a specific resolution [9]. Another advantage of the Wavelet technique is that various Wavelet functions allow selecting the best function for analyzing the signal, whereas in the case of Fourier analysis, it is restricted to only one feature morphology that is the sinuous [10].

For a function $f(t) \in L^2(\mathbb{R})$ which is a one-dimensional vector space of functions which are measurable and integrable in a sense of the mean square, the continuous Wavelet transform (CWT) is given by [11] as:

$$
\begin{aligned}
CWT_f^\psi(\tau, s) &= \int f(t)\psi_{\tau,s}^*(t)dt, \\
\psi_{\tau,s}(t) &= \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right),
\end{aligned}
\tag{1}
$$

where ψ^* is the complex conjugate of the wavelet, 's' is a scale parameter which can be converted to frequency ($\frac{1}{s}$ reflects the frequency) and 'τ' a shift parameter represents the location of the wavelet along the time axis.

The CWT is a correlation between a wavelet at different scales and the signal $f(t)$ (the data to be analyzed) with the scale (or the frequency) being used as a measure of similarity [12]. It can be computed by changing the scale of the analysis window function by shifting the window function in time, multiplying by the signal, and integrating over all times. It is an efficient method of providing localization of a signal in both time and frequency domain by time scale representation of continuous time signals. The concept of a continuous

wavelet transform (CWT) requires the continuous scale and continuous shift in time.

In this study, Wavelet analysis of ECG signals are exemplified with MIT-BIH Arrhythmia Database which is a freely available resource intended for studies and research. It contains 48 half-hour excerpts of two-channel ambulatory ECG recordings, obtained from 47 subjects studied by the BIH Arrhythmia Laboratory. The recordings were digitized at 360 samples per second per channel with 11-bit resolution over 10 mV range. The records are available in many formats. The user can have the option of the signal duration, the format of the time, and data [13].

Methodology

Wavelets and all their variants result in two dimensional continuous and discretized outputs of two parameters namely the scale and time shift. While the time shift corresponds to time axis of the signal to be analyzed, scale corresponds but is not equal to frequency. However, in wavelet analysis of a given signal, we have to take into account the type of the Wavelet transform, type of the mother wavelet and the desired scale. An infinite number of options are available in wavelet analysis involving the above aspects which are closely related and must be considered at the same time. The method described in this study is based on continuous wavelet transform (CWT) for the following reasons:

- The discrete wavelet transform (DWT) could lose frequency resolution due to resampling at each decomposition level [14], but the CWT ensures a good frequency resolution.

- With CWT, the appropriate and dominant scale for each component of ECG signal can be extracted, which makes it possible to detect each component separately [15].

As there is no predefined criteria to choose a mother wavelet for a particular application, Aqil et al. in [7] developed a criteria based on the inequality of Cauchy-Schwartz. Applying Cauchy-Schwartz inequality to equation (1) gives

$$CWT_f^\psi(\tau, s) = |\langle f, \psi(\tau, s)\rangle| \le ||f|| \, ||\psi||$$

$$\implies \left| \int_{-\infty}^{\infty} f(t)\psi_{\tau,s}^*(t)dt \right| \le \int_{-\infty}^{\infty} |f(t)|^2 dt . \int_{-\infty}^{\infty} |\psi_{\tau,s}^*(t)|^2 dt, \tag{2}$$

where $||f||$ is the energy of the signal and $||\psi||$ is the energy of the wavelet.

Equation (2) implies the colinearity of $f(t)$ and $\psi(t)$, and can be defined in the following way

$$L = \frac{|\int_{-\infty}^{\infty} f(t)\psi_{\tau,s}^*(t)dt|}{\int_{-\infty}^{\infty} |f(t)|^2 dt . \int_{-\infty}^{\infty} |\psi_{\tau,s}^*(t)|^2 dt}, \tag{3}$$

and the discrete form of the equation (3) is given by

$$L = \frac{\sum_{n=1}^{N}(\sum_{k=1}^{K} |C_f(n, s_k)|^2)}{\sum_{n=1}^{N} |f(n)|^2 . \sum_{n=1}^{N} |\psi(n)|^2}, \tag{4}$$

where $C_f(n, s_k)$ is the CWT coefficients, N is the number of samples, K is the number of scales, $f(n)$ is the sample of signal $f(n)$ at time t_n, and $\psi(n)$ is the sample of ψ function of the wavelet. Colinearity L was used by the authors of [7] as a criteria for the choice of mother wavelet and concluded that Morlet is the best suited for their application because of higher L value. Further, this selection criteria can be also examined based on the statistical parameters such as auto and cross correlation.

In this study, we propose three mother wavelets namely Mxr-1, Mxr-2 and Mxr-3 which are an average, half difference and product of Mexican hat and Morlet wavelets respectively. They are defined as:

$$\psi_{(Mxr-1)}(t) = e^{\frac{-t^2}{2}}[\frac{1}{\sqrt{3}\pi^{1/4}}(1 - t^2) + \frac{1}{2}\cos(5t)], \tag{5}$$

$$\psi_{(Mxr-2)}(t) = e^{\frac{-t^2}{2}}[\frac{1}{\sqrt{3}\pi^{1/4}}(1 - t^2) - \frac{1}{2}\cos(5t)], \tag{6}$$

$$\psi_{(Mxr-3)}(t) = \frac{2}{\sqrt{3}\pi^{1/4}} e^{-t^2} (1 - t^2) \cos(5t). \tag{7}$$

The proposed wavelets along with Morlet, Mexican hat and Db4 wavelets are shown in Figure 2. The proposed wavelets qualify to be standard wavelets as they satisfy the conditions of zero mean ($\int_{-\infty}^{\infty} \psi(t) = 0$) and compact support. The representation of ECG signal (one cycle of record 115) with different mother wavelet shapes are shown in Figure 3. In addition, cross correlation of ECG signal (record 100) and Mxr-1 wavelet are illustrated in Figure 4.

Implementation of the method to select a particular scale for R peaks detection in ECG signal requires that first we compute continuous wavelet transform $C(n, s_k)$ up to 32 scale with the proposed mother wavelet Mxr-1 of a record '115' from the MIT-BIH database. Then, we extract the maximum wavelet coefficients of $C(n, s_k)$ at each time point and denote them as (x_m, s_m) where

x_m is the maximum CWT coefficient and s_m is the corresponding scale. Once this is complete, we pick up the peaks of x_m and look for all the scales corresponding to these peaks. Then, estimate the arithmetic average of the scales corresponding to the peaks of maximum CWT coefficients which we denote as s_R. Then, we apply the moving average filter to reduce random noise while retaining a critical deflection response. Finally, we find the peaks in a denoised selected scale (denoised s_R) and coincide with the R peaks in the original ECG signal to verify the validity of these peaks to be R-peaks. Further detail in detection of R peaks can be seen [7]. The process is applied to other records (100, 101, 103, 112, 113, 117, 122, 123) from the same database. For comparison, we apply CWT with different mother wavelets such as Morlet, Mexican hat, Db4, Mxr-2 and Mxr-3 for the same records. The steps of the computation are given in Figure 5. The R-peaks selected based on the procedure outlined by Aqil et al. in [7] may be validated by three parameters such as sensitivity (Se), positive predictivity (P+), and detection error (DER) that are given by the following relations [16]:

$$Se = \frac{TP}{TP + FN} \times 100\%, \tag{8}$$

$$P+ = \frac{TP}{TP + FP} \times 100\%, \tag{9}$$

$$DER = \frac{FP + FN}{TP} \times 100\%, \tag{10}$$

where TP is the true positive (correct detection of R peaks), FN is false negative (undetected R peaks), and FP is false positive (misdetections).

Results and Discussion

The ECG record 115 of lead L2 from MIT-BIH database are shown in Figure 6(a) where the maximum and corresponding scales of continuous wavelet coefficients based on the proposed Mxr-1 wavelet are shown in Figure 6(b) and 6(c) respectively. In Figure 7, the absolute of noisy CWT of a selected scale of the record 115 is shown. The comparison of peaks in denoised selected scale with the R-peaks in the original signal are illustrated in Figure 8. The detected R-peaks by different mother wavelets such as Morlet, Mexican hat, Db4 and
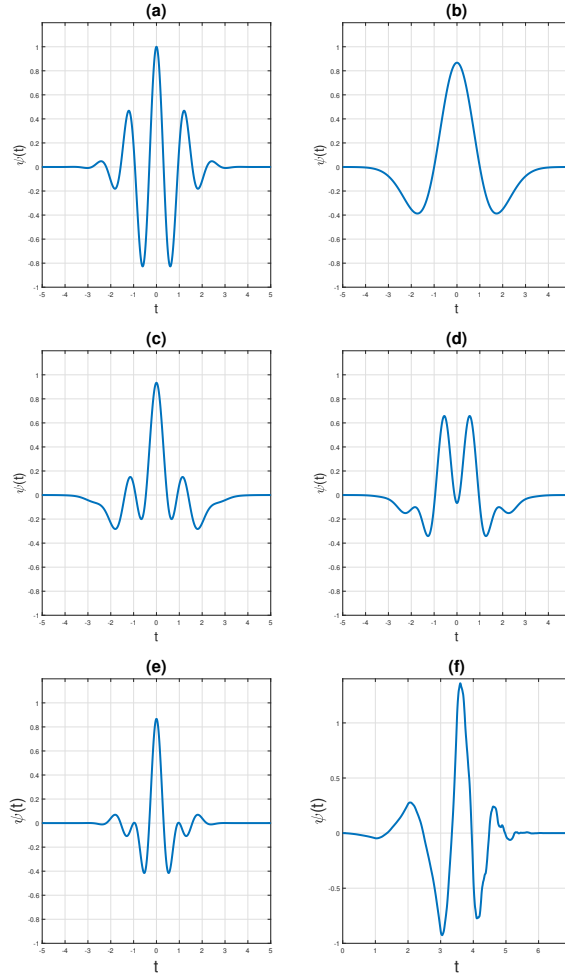
Figure 2: Standard and proposed mother wavelets: a) Morlet wavelet b) Mexican hat wavelet c) Mxr-1 wavelet d) Mxr-2 wavelet e) Mxr-3 wavelet f) db4 wavelet.

the proposed wavelets (Mxr-1, Mxr-2, Mxr-3) on different records (100, 101, 103, 112, 113, 117, 122, 123) from MIT-BIH Arrhythmia Database are shown in Tables 2 and 3. The combined average of sensitivity parameters for all the considered records by different wavelets are shown in Table 4. The values of colinearity and value of auto correlation at zeroth lag of different wavelets are given in Table 5. In addition, Table 5 also include the combined average of computed minimum and maximum values of cross correlation of wavelets for

Figure 3: Representation of ECG waveform (record 115) and mother wavelets given in Figure (2)

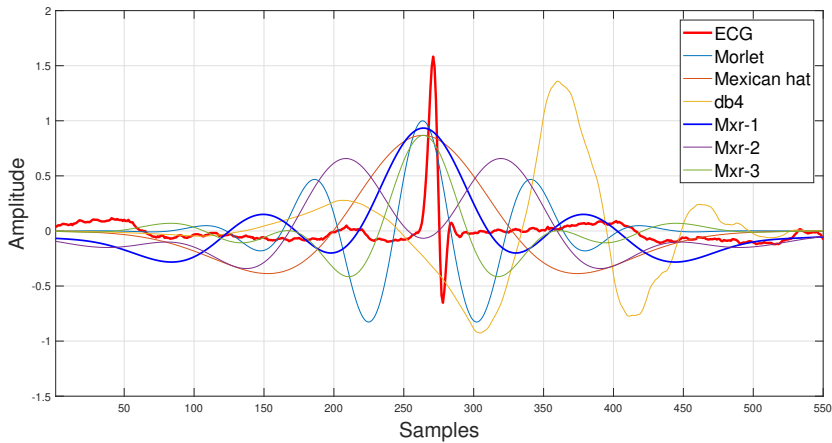all data sets (100, 101, 103, 112, 113, 117, 122, 123). The performance of CWT with one of the proposed wavelets Mxr-1 achieves 99.97 % sensitivity, 99.89% positive predictivity, and 0.135 % detection error for accurate detection of R peaks with the a marginal improvement in comparison with the well known wavelets such as Morlet, Mexican hat and Db4, and the other two proposed wavelets namely, Mxr-2 and Mxr-3. Although the colinearity, auto correlation and range of cross correlation of Mxr-1 are not the highest of all the wavelets studied including the well known wavelets, the performance of the proposed wavelet Mxr-1 is slightly better in detection of R peaks. We can thus conclude that there is no obvious relation between the performance of wavelet in detecting R peaks with colinearity and statistical parameters like auto and cross correlation.

Conclusion

The accuracy and reliability of detection of R-peaks based on the proposed wavelet Mxr-1 yields 99.97 % sensitivity, 99.89 % positive predictivity, and 0.135 % detection error which is marginally higher than those obtained based on the Morlet, Mexican hat, Db4 and other proposed wavelets. This substantiates the applicability of the proposed Mxr-1 wavelet in electrocardiogram signal analysis. Further, it can be elucidated that the choice of the wavelet in such
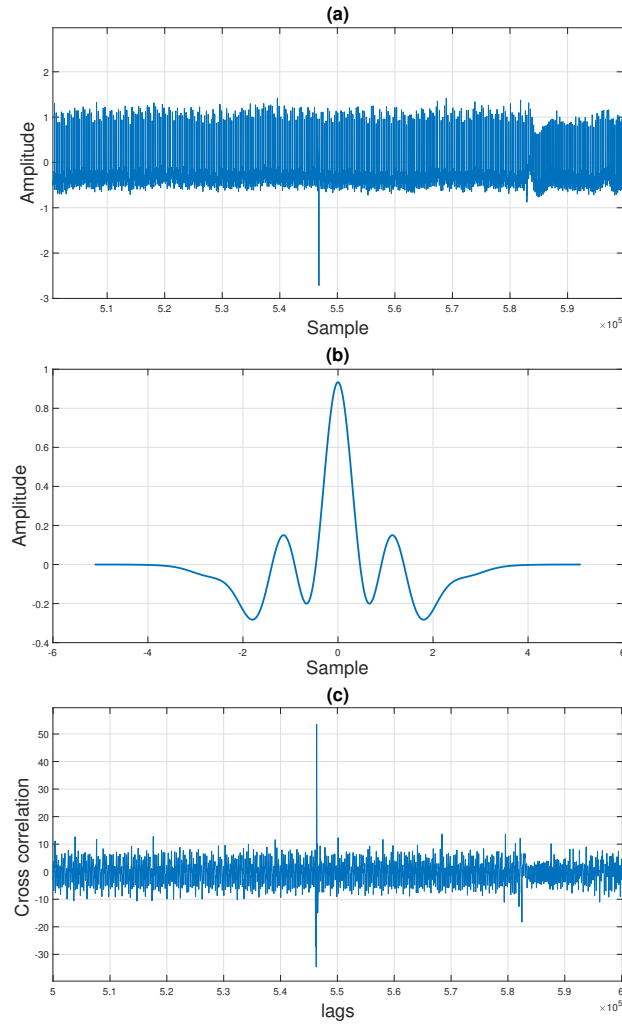
Figure 4: Representative illustration of ECG signal (record 100), Mxr-1 wavelet and their cross correlation

an application is purely based on trial and error basis, although colinearity and statistical parameters such as auto and cross correlation may support to limited extent. Incidentally, the proposed wavelet Mxr-1 result the maximum cross correlation with ECG signal (record 100) exactly over the minimum of the signal which is an interesting feature in this study.

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┌──────────────────────────────────────────┐
│     Reading and normalizing the ECG signal │
└──────────────────────────────────────────┘
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│     Localizing the maximum CWT coefficients │
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│  Automatic selection of a scale corresponding │
│  to a maximum energy                        │
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│  Maximum wavelet analysis at a selective scale │
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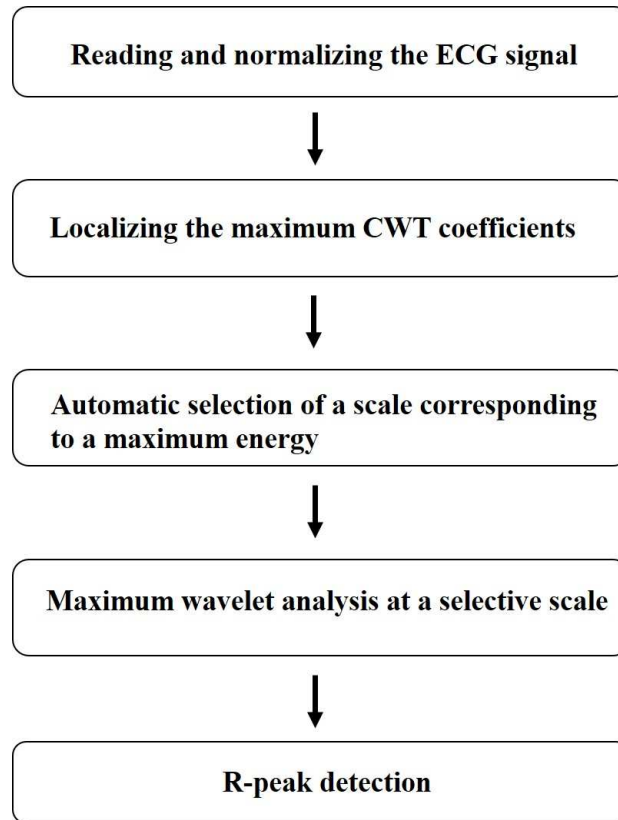
Figure 5: Flow chart of computation of R peaks

References

[1] D. A. Coast, R. M. Stern, G. G. Cano, S. A. Briller, An approach to cardiac arrhythmia analysis using hidden Markov models, *IEEE Trans. Biomed. Eng.*, **37** (1990), 826-836.

[2] Y. H. Hu, W. J. Tompkins, J. L. Urrusti, V. X. Afonso, Applications of artificial neural networks for ECG signal detection and classification, *J. Electrocardio.*, **26**, (1993), 66-73.

[3] R. Poli, S. Cagnoni, G. Valli, Genetic design of optimum linear and non-linear QRS detectors, *IEEE Trans. Biomed. Eng.*, **42** (1995), 1137-1141.

[4] D. Benitez, P .Gaydecki, A. Zaidi, A. Fitzpatrick, The use of the Hilbert transform in ECG signal analysis, *Comput. Biol. Med.*, **31** (2001), 399-406.
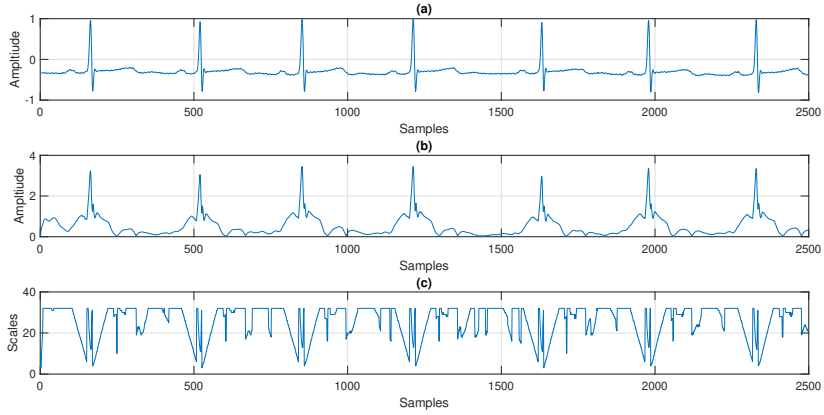
Figure 6: a) ECG signal of record 115 from MIT-BIH database.
b) Maximum of CWT coefficient of ECG signal with Mxr-1 wavelet.
c) Scales corresponding to the maximum of CWT coefficient of ECG
signal with Mxr-1 wavelet.

Figure 7: Absolute of noisy CWT coefficients for a selected scale of
ECG signal based on Mxr-1 wavelet.

[5] G. Vega-Martnez, C. Alvarado-Serrano, L. Leija-Salas, Wavelet packet
based algorithm for QRS region detection and R/S wave identification,
IEEE Conference publications., (2015), 1-6.

Figure 8: Correlation of peaks of selected denoised scale from CWT coefficients (top) with the original (record 115) ECG signal (bottom).

[6] S. Kadambe, R. Murray, G. F. Boudreaux-Bartels, Wavelet transform based QRS complex detector, *IEEE Trans. Biomed. Eng.*, **46** (1999), 838-848.
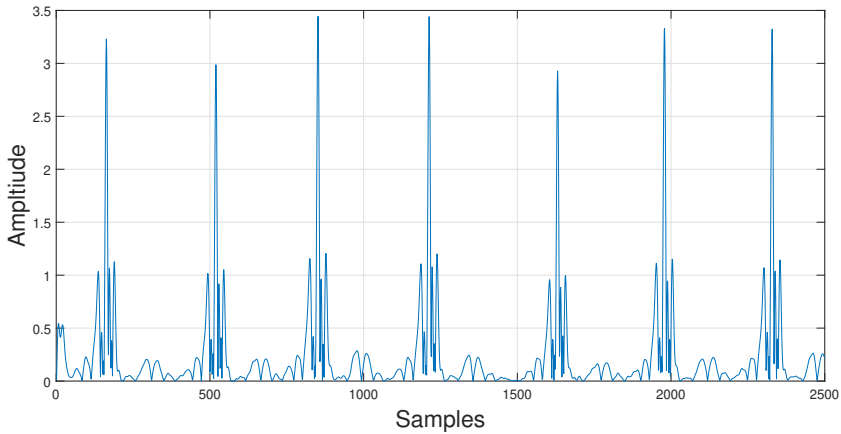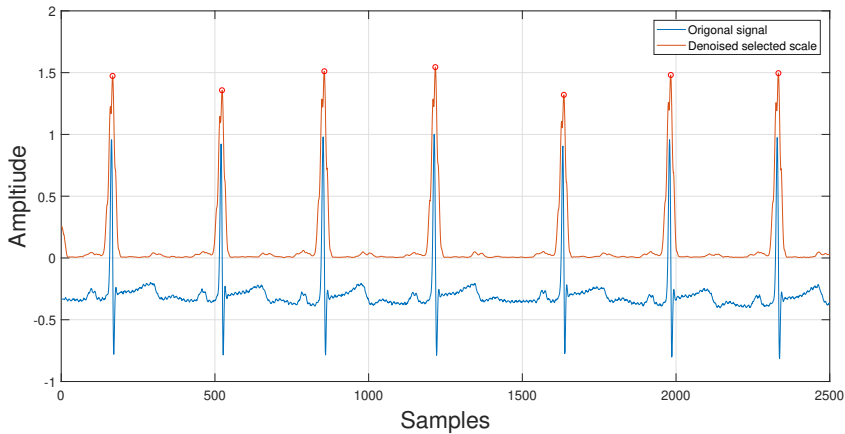
[7] M. Aqil, A. Jbari, A.Bourouhou, ECG-waves: analysis and detection by continuous wavelet transform, *J. Telecommun. Electron. Comput. Eng.*, **9** (2017), 45-52.

[8] M. Alfaouri, K. Daqrouq, ECG signal denoising by wavelet transform thresholding, *Am. J. Appl. Sci.*, **5** (2008), 276-281.

[9] Priyanka Mehta, Monika Kumari, QRS Complex Detection of ECG Signal Using Wavelet Transform, *Int. J. Appl. Eng. Res.*, **7** (2012), 1889-1893.

[10] Paul. S. Addison, Wavelet Transforms and the ECG: a review, *Physiol. Meas.*, **26** (2005), R155-R199.

[11] A. Grossmann, J. Morlet, Decomposition of Hardy functions into square integrable wavelets of constant shape, *SIAM J. Math. Anal.*, **15** (1984), 723-736.

[12] R. Polikar, The wavelet tutorial, (1996), https://cseweb.ucsd.edu/ baden/Doc/wavelets/polikar_wavelets.pdf.

[13] Physiont, MIT-BIH Arrhythmia Database, https://physionet.org/content/mitdb/1.0.0/

Table 2: Comparison of detection of R peaks by different wavelets for the records 100, 101, 103, 112, 113

Record	Wavelet	Detected beats	TP	FP	FN	Se%	P+	DER
100	Morlet	2274	2273	1	0	100	99.96	0.044
	Mexican hat	2274	2273	1	0	100	99.96	0.044
	Db4	2274	2273	1	0	100	99.96	0.044
	Mxr-1	2274	2273	1	0	100	99.96	0.044
	Mxr-2	2274	2273	1	0	100	99.96	0.044
	Mxr-3	2274	2273	1	0	100	99.96	0.044
101	Morlet	1876	1860	16	0	100	99.15	0.853
	Mexican hat	1873	1860	13	0	100	99.31	0.694
	Db4	1868	1859	9	0	100	99.52	0.482
	Mxr-1	1866	1858	7	0	100	99.62	0.375
	Mxr-2	1866	1859	7	0	100	99.62	0.375
	Mxr-3	1866	1859	7	0	100	99.62	0.375
103	Morlet	2086	2084	2	0	100	99.90	0.096
	Mexican hat	2086	2084	2	0	100	99.90	0.096
	Db4	2086	2084	2	0	100	99.90	0.096
	Mxr-1	2085	2084	1	0	100	99.95	0.048
	Mxr-2	2085	2084	1	0	100	99.95	0.048
	Mxr-3	2085	2084	1	0	100	99.95	0.048
112	Morlet	2539	2539	0	0	100	100	0
	Mexican hat	2539	2539	0	0	100	100	0
	Db4	2539	2539	0	0	100	100	0
	Mxr-1	2539	2538	1	0	100	99.96	0.039
	Mxr-2	2539	2529	10	0	100	99.61	0.394
	Mxr-3	2539	2537	2	0	100	99.92	0.079
113	Morlet	1798	1790	8	0	100	99.56	0.445
	Mexican hat	1794	1787	7	1	99.94	99.61	0.446
	Db4	1791	1786	5	4	99.78	99.72	0.503
	Mxr-1	1798	1790	8	0	100	99.56	0.445
	Mxr-2	1794	1789	5	1	99.94	99.72	0.334
	Mxr-3	1798	1790	8	0	100	99.56	0.445

[14] M. Yochum, Ch. Renaud, S. Jacquir, Automatic detection of P, QRS, and T patterns in 12 leads ECG signal based on CWT, *Biomed. Signal. Process.*

Table 3: Comparison of detection of R peaks by different wavelets for the records 115, 117, 122, 123

Record	Wavelet	Detected beats	TP	FP	FN	Se%	P+	DER
115	Morlet	1953	1953	0	0	100	100	0
	Mexican hat	1953	1953	0	0	100	100	0
	Db4	1952	1952	0	1	99.95	100	0.051
	Mxr-1	1953	1953	0	0	100	100	0
	Mxr-2	1953	1950	3	0	100	99.85	0.154
	Mxr-3	1953	1953	0	0	100	100	0
117	Morlet	1533	1524	9	2	99.87	99.41	0.718
	Mexican hat	1534	1534	0	1	99.93	100	0.065
	Db4	1534	1534	0	1	99.93	100	0.065
	Mxr-1	1534	1534	0	1	99.93	100	0.065
	Mxr-2	1533	1532	1	2	99.87	99.93	0.196
	Mxr-3	1534	1533	1	1	99.93	99.93	0.130
122	Morlet	2476	2474	2	0	100	99.92	0.081
	Mexican hat	2476	2476	0	0	100	100	0
	Db4	2476	2476	0	0	100	100	0
	Mxr-1	2476	2476	0	0	100	100	0
	Mxr-2	2476	2476	0	0	100	100	0
	Mxr-3	2475	2475	0	1	99.96	100	0.040
123	Morlet	1513	1509	4	5	99.67	99.74	0.595
	Mexican hat	1515	1515	0	3	99.80	100	0.198
	Db4	1515	1515	0	3	99.80	100	0.198
	Mxr-1	1515	1515	0	3	99.80	100	0.198
	Mxr-2	1513	1512	1	5	99.67	99.93	0.397
	Mxr-3	1515	1515	0	3	99.80	100	0.198

Control., **25** (2016), 46-52.

[15] A. Ghaffari, H. Golbayani, M. Ghasemi, A new mathematical based QRS detector using continuous wavelet transform, *Comput. Electr. Eng.*, **34** (2008), 81-91.

[16] I. Kaur, R. Rajni, A. Marwaha, ECG signal analysis and arrhythmia detection using wavelet transform, *J. Inst. Eng. (India)*, **97** (2016), 499-507.

Table 4: Combined average of sensitivity parameters for different wavelets

Wavelets	Se%	P+	DER
Morlet	99.95	99.74	0.315
Mexican hat	99.96	99.86	0.171
Db4	99.94	99.90	0.160
Mxr-1	99.97	99.89	0.135
Mxr-2	99.94	99.84	0.216
Mxr-3	99.97	99.88	0.151

Table 5: Colinearity and statistical parameters for different wavelets

Wavelets	Colinearity	Auto correlation (max value)	Cross correlation (min)	(max)
Morlet	0.5152	56.67	-23.98	24.06
Mexican hat	1.3202	63.94	-58.27	49.59
Db4	0.0341	1024	-657.53	586.65
Mxr-1	1.2085	48.16	-56.48	46.33
Mxr-2	2.4158	46.157	-56.87	47.40
Mxr-3	0.8525	32.47	-27.85	24.79

[17] I. Romero Legarreta1, P. S. Addison, N. R. Grubb, G. R. Clegg, C. E. Robertson, J. N. Watson, A comparison of continuous wavelet transform and modulus maxima analysis of characteristic ECG features, *Comput. Cardiol.*, **32** (2005), 755-758.