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APPLICATIONS OF FUZZY SETS FOR ALMOSTITY OF TERNARY SUBSEMIRINGS

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Abstract: In this paper, we introduce almost ternary subsemirings of ternary semirings. We also investigate the properties of the union and the intersection of two almost ternary subsemirings. Moreover, we apply the fuzzy sets to study the fuzzifications of almost ternary subsemirings and provide the relationships among almost ternary subsemirings and their fuzzifications.

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1. Introduction

In 1932, Lehmer [14] introduced the notion of ternary algebraic structure, but earlier such structure was studied by Kasner [9] in 1904 and Prüfer [18] in 1924. Lehmer investigated certain ternary algebraic system called triplexes. In 1971, Lister [15] characterized additive semigroups of rings which are closed under the triple ring product and it is called a ternary ring. The structure of ternary semiring is generalization of the structures of rings and semirings. Moreover, ternary semirings under triple ring product will be a ternary semigroups. Then

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the study on ternary semirings will generalize outputs of rings and semirings. Rao [19] defined another basic properties of ternary semirings and characterized them. Ideal theory of ternary semirings were studied in [4], [5], [7], [10] and [22]. Moreover, Dutta and Kar [6] studied a notion of regular ternary semirings. Fuzzy sets were introduced by Zadeh [23] in 1965 as an extension of the classical notion of set. Fuzzy set theory permits the gradual assessment of the membership of elements in a set; this is described with the aid of a membership function valued in the unit interval $[0, 1]$. Applications of fuzzy subsets have been developed in many fields. In this communication, we will focus on applications of fuzzy sets in algebraic structures. Rosenfeld applied the fuzzy subsets to define fuzzy subgroups of groups in [21]. Later, there are many researchers that apply fuzzy sets to study the fuzzifications of some concepts of many algebraic structures. Fuzzy sets were generalized to other concepts. In addition, fuzzy sets were studied in the structures of ternary semirings in [17], [1], [3], [16] and [11].

The concept of almost subsemigroups of semigroups and their fuzzifications [8] were first introduced in 2021. After that the characterization of almost ternary subsemigroups and their fuzzifications were presented in [2]. Recently, the almost subsemirings and fuzzifications were studied in [20].

In this paper, we will recall some basic notations and definitions in Section 2. The main results of this paper are included in Section 3. Our purposes are the following:

(1) To introduce the notions of almost ternary subsemirngs and investigate their basic properties.

(2) To apply fuzzy set in order to introduce almost ternary subsemiring and study their properties.

(3) To investigate the relationship between almost ternary subsemirings and their fuzzifications.

2. Preliminaries

The purpose of this section is to recall some basic notations and definitions used throughout this article. These can be also found in [8, 19, 20].

2.1. Ternary semirings

Let T be a nonempty set. The set T together with a binary operation known as addition and a ternary multiplication is said to be a ternary semiring if $(T, +)$ is a commutative semigroup satisfying the following conditions:

(i) $(abc)de = a(bcd)e = ab(cde)$,

(ii) $(a + b)cd = acd + bcd$,

(iii) $a(b + c)d = abd + acd$,

(iv) $ab(c + d) = abc + abd$,

for all $a, b, c, d, e \in T$.

An additive subsemigroup A of a ternary semiring T is called a ternary subsemiring of S if $abc \in A$ for all $a, b, c \in A$.

Throughout this paper, we denote a ternary semiring by T unless otherwise stated.

2.2. Fuzzy subsets

We recall that a fuzzy subset of a set S is a membership function from S into the closed interval $[0, 1]$. Let f and g be any two fuzzy subsets of S. We define fuzzy subsets $f \cup g$ and $f \cap g$ of S by

$$(f \cup g)(x) = \max\{f(x), g(x)\}$$

and

$$(f \cap g)(x) = \min\{f(x), g(x)\}$$

for all $x \in S$. If $f(x) \leq g(x)$ for all $x \in S$, we say that $f \subseteq g$.

Let f be a fuzzy subset of S. The support of f is defined by

$$supp(f) = \{x \in S \mid f(x) \neq 0\}.$$

Let A be a subset of S. The characteristic mapping of A is a fuzzy subset C_A of S defined by

$$C_A = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{otherwise.} \end{cases}$$

For any three fuzzy subsets f, g and h of a ternary semiring T, fuzzy subsets $f + g$ and $f \circ g \circ h$ are defined as follows:

For any $x \in T$,

$$(f+g)(x) = \sup_{x=a+b} \{\min\{f(a), g(b)\}\}$$

if $x = a+b$ for some $a, b \in T$. Otherwise, $(f+g)(x) = 0$. And for any $x \in T$,

$$(f \circ g \circ h)(x) = \sup_{x=abc} \{\min\{f(a), g(b), h(c)\}\}$$

if $x = abc$ for some $a, b, c \in T$. Otherwise, $(f \circ g \circ h)(x) = 0$.

Let f be a fuzzy subset of a ternary semiring T. If $f+f \subseteq f$ and $f \circ f \circ f \subseteq f$, then T is called a fuzzy ternary subsemiring of T.

3. Main results

We extend the concepts of almostity to define almost ternary subsemirings of ternary semigroups in Subsection 3.1 and investigate their basic properties. Moreover, we apply the concepts of fuzzy sets to study the fuzzifications of almost ternary subsemirings in Subsection 3.2. In addition, the investigation of the relationship between almost ternary subsemirings and their fuzzifications is also presented in Subsection 3.2.

3.1. Almost ternary subsemirings

The goal of this section is to present the definition of an almost ternary subsemiring and study its property.

Definition 1. Let A be a nonempty set of a ternary semiring T. Then the set A is called an *almost ternary subsemiring* of T if $(A+A) \cap A \neq \emptyset$ and $A^3 \cap A \neq \emptyset$.

Let A be any ternary subsemiring of a ternary semiring T. Then $A+A \neq \emptyset, A^3 \neq \emptyset$, $A+A \subseteq A$, and $A^3 \subseteq A$. It follows that $(A+A) \cap A \neq \emptyset$ and $A^3 \cap A \neq \emptyset$. Thus A is an almost ternary subsemiring of T. Hence, we can say that every ternary subsemiring of a ternary semiring T is also an almost ternary subsemiring of T.

Example 2. Consider a ternary semiring $(\mathbb{Z}^-, +, \cdot)$. Let

$$A = \{-2, -4, -8\} \text{ and } B = \{-1, -2, -8\}.$$

It is obvious that A and B are almost ternary subsemirings but are not ternary subsemirings of $\mathbb{Z}^-$. However, $A \cap B = \{-2, -8\}$ is not an almost ternary subsemiring of $\mathbb{Z}^-$.

In general, we obtain the following facts:
(1) An almost ternary subsemiring of a ternary semiring T does not need to be a ternary subsemiring of T.
(2) The intersection of almost ternary subsemirings of a ternary semiring T does not need to be an almost ternary subsemiring of T.

Proposition 3. *Let a be any element of a ternary semiring T. Then $A = \{a, a+a, a^3\}$ is an almost ternary subsemiring of T.*

Proof. Let a be any element of a ternary semiring T. Then $a + a \in (A + A) \cap A$ and $a^3 \in A^3 \cap A$. Therefore $A = \{a, a+a, a^3\}$ is an almost ternary subsemiring of T. □

For any elements a, b of a ternary semiring T, we can see that $A = \{a, b, a+a, a^3\}$ is an almost ternary subsemiring of T because $a + a \in (A + A) \cap A$ and $a^3 \in A^3 \cap A$. There are some other almost ternary subsemirings of T generated from a, b such as $\{a, b, a+a, b^3\}, \{a, b, a+b, a^3\}$, and $\{a, b, a+b, b^3\}$.

Proposition 4. *Let a be any element of a ternary semiring T. The set $\{a\}$ is an almost ternary subsemiring of T if and only if $a = a + a$ and $a^3 = a$.*

Proof. The proof is obvious and omitted for brevity. □

Theorem 5. *Let A and B be any two nonempty subsets of a ternary semiring T. If $A \subseteq B$ and A is an almost ternary subsemiring of T, then B is also an almost ternary subsemiring of T.*

Proof. We assume that A is an almost ternary subsemiring of a ternary semiring T. Then $(A + A) \cap A \neq \emptyset$ and $A^3 \cap A \neq \emptyset$. Since $A \subseteq B$, there exist $a, b, x, y, z \in A$ such that $a + b \in (B + B) \cap B$ and $xyz \in B^3 \cap B$. It follows that $(B + B) \cap B \neq \emptyset$ and $B^3 \cap B \neq \emptyset$. Hence B is an almost ternary subsemiring of T. □

From Theorem 5, we obtain the following corollaries:

Corollary 6. *Let n be a positive integer and $A_i, i \in I = \{1, 2, \ldots, n\}$ be almost ternary subsemirings of a ternary semiring T. The union $\cup_{i\in I} A_i$ is also an almost ternary subsemiring of T.*

Corollary 7. *Let A and B be any two nonempty subsets of a ternary semiring T. If A is an almost ternary subsemiring of T, then $A \cup B$ is also an almost ternary subsemiring of T.*

Corollary 8. *Let n be a positive integer and $A_i, i \in I = \{1, 2, \ldots, n\}$ be nonempty subset of T. The union $\cup_{i\in I} A_i$ is an almost ternary subsemiring of T if there exists an almost ternary subsemiring A_i for some $i \in I$ in the union.*

3.2. Fuzzy almost ternary subsemirings

In this section, we study the concept of fuzzy almost ternary subsemirings and its properties.

Definition 9. A fuzzy subset of a ternary semiring T is called a *fuzzy almost ternary subsemiring* of T if $(f + f) \cap f \neq 0$ and $(f \circ f \circ f) \cap f \neq 0$.

Let f be any fuzzy ternary subsemiring of a ternary semiring T such that $f + f \neq 0$ and $f \circ f \circ f \neq 0$. Then $f + f \subseteq f$ and $f \circ f \circ f \subseteq f$. It follows that $(f + f) \cap f \neq 0$ and $(f \circ f \circ f) \cap f \neq 0$. Hence, f is a fuzzy almost subsemiring of T.

Example 10. Consider the semiring $(\mathbb{Z}^-, +, \cdot)$. Let f and g be the following fuzzy subsets of $\mathbb{Z}^-$:

$$f(x) = \begin{cases} 0.3 & \text{if } x \in \{-1, -2\}, \\ 0 & \text{otherwise}, \end{cases}$$

and

$$g(x) = \begin{cases} 0.2 & \text{if } x \in \{-2, -4, -8\}, \\ 0 & \text{otherwise}. \end{cases}$$

We can say that f and g are fuzzy almost ternary subsemirings but are not fuzzy ternary subsemirings. However, $f \cap g$ is not a fuzzy almost ternary subsemiring of $\mathbb{Z}^-$.

In general, we obtain the following facts:

(1) Any fuzzy almost ternary subsemiring of a ternary semiring T does not need to be a fuzzy ternary subsemiring of T.
(2) The intersection of fuzzy almost ternary subsemirings of a semiring T does not need to be a fuzzy almost ternary subsemiring of T.

Theorem 11. *Let f and g be fuzzy subsets of a ternary semirings T such that $f \subseteq g$. If f is a fuzzy almost ternary subsemiring of T, then g is also a fuzzy almost ternary subsemiring of T.*

Proof. Suppose that f is a fuzzy almost ternary subsemiring of a ternary semiring T. Then $(f+f) \cap f \neq 0$ and $(f \circ f \circ f) \cap f \neq 0$. It follows that $(f+f) \cap f \subseteq (g+g) \cap g$ and $(f \circ f \circ f) \cap f \subseteq (g \circ g \circ g) \cap g$ because $f \subseteq g$. It implies that $(g+g) \cap g \neq 0$ and $(g \circ g \circ g) \cap g \neq 0$. Hence, g is a fuzzy almost ternary subsemiring of T. □

Corollary 12. *Let n be a positive integer and $f_i, i \in I = \{1, 2, \ldots, n\}$ be fuzzy almost ternary subsemirings of a ternary semiring T. The union $\cup_{i \in I} f_i$ is also a fuzzy almost ternary subsemiring.*

Corollary 13. *Let f and g be fuzzy subsets of a ternary semirings. If f is a fuzzy almost ternary subsemiring of T, then $f \cup g$ is also a fuzzy almost ternary subsemiring of T.*

Corollary 14. *Let n be a positive integer and $f_i, i \in I = \{1, 2, \ldots, n\}$ be fuzzy subsets of a ternary semiring T. The union $\cup_{i \in I} f_i$ is a fuzzy almost ternary subsemiring if there exists a fuzzy almost ternary subsemiring f_i for some $i \in I$ in the union.*

Proposition 15. *Let A, B be any subsets of a ternary semiring T. Then $C_{A \cup B} = C_A \cup C_B$.*

Proof. Let $x \in T$. If $x \in A \cup B$, then $C_{A \cup B}(x) = 1$ and $(C_A \cup C_B)(x) = \max\{C_A(x), C_B(x)\} = 1$. If $x \notin A \cup B$, then $C_{A \cup B}(x) = 0$ and $(C_A \cup C_B)(x) = \max\{C_A(x), C_B(x)\} = 0$. Thus $C_{A \cup B} = C_A \cup C_B$. □

Theorem 16. *Let A be a nonempty subset of a ternary semiring T. Then A is an almost ternary subsemiring of T if and only if C_A is a fuzzy almost ternary subsemiring of T.*

Proof. Suppose that A is an almost ternary subsemiring of a ternary semiring T. It follows that $(A + A) \cap A \neq \emptyset$ and $A^3 \cap A \neq \emptyset$. Thus there exists $x \in A^3 \cap A$. Hence $x = abc$ for some $a, b, c \in A$ and $x \in A$. We now have $(C_A \circ C_A \circ C_A)(x) = 1$ and $C_A(x) = 1$. This implies that $[(C_A \circ C_A \circ C_A) \cap C_A](x) = 1 \neq 0$ which means $(C_A \circ C_A \circ C_A) \cap C_A \neq 0$. We can similary show that $(C_A + C_A) \cap C_A \neq 0$. Therefore C_A is a fuzzy almost ternary subsemiring of T.

Conversely, suppose that C_A is a fuzzy almost ternary subsemiring of a ternary subsemiring T. Then $(C_A + C_A) \cap C_A \neq 0$ and $(C_A \circ C_A \circ C_A) \cap C_A \neq 0$. Thus there exists $x \in T$ such that $[(C_A \circ C_A \circ C_A) \cap C_A)](x) \neq 0$. It follows that $(C_A \circ C_A \circ C_A)(x) = 1$ and $C_A(x) = 1$. Hence $x \in A^3$ and $x \in A$. We now have $A^3 \cap A \neq \emptyset$. Therefore A is an alomost ternary subsemiring of T. □

Proposition 17. *Let A, B be any subsets of a ternary semiring T. If A is an almost ternary subsemiring of T, then $C_A \cup C_B$ is a fuzzy almost ternary subsemiring of T.*

Proof. By Corollary 13, Proposition 15 and Theorem 16. □

The following example shows that for any subsets A, B of a ternary semiring T, if $C_A \cup C_B$ is a fuzzy almost ternary subsemiring of T, then C_A and C_B do not need to be fuzzy almost ternary subsemirings.

Example 18. Consider a ternary $(\mathbb{Z}, +, \cdot)$. Let $A = \{-1, -8\}$ and $B = \{-2, -4\}$. We can see that A and B are not almost ternary subsemirings, but $A \cup B$ is an almost ternary subsemiring. By Theorem 16 and Proposition 17, $C_A \cup C_B$ is a fuzzy almost ternary subsemirings. However, C_A and C_B are not fuzzy almost ternary subsemirings.

Proposition 19. *Let f, g be any fuzzy subsets of a ternary semiring T. Then $supp(f \cup g) = supp(f) \cup supp(g)$.*

Proof. Let $x \in T$. If $x \in supp(f \cup g)$, then $(f \cup g)(x) \neq 0$. It implies $(f \cup g)(x) = \max\{f(x), g(x)\} \neq 0$. Without loss of generality, we assume that $f(x) \geq g(x)$. We now have $(f \cup g)(x) = f(x) \neq 0$ and $x \in supp(f)$. Hence $x \in supp(f) \cup supp(g)$. It implies that $supp(f \cup g) \subset supp(f) \cup supp(g)$. We will now show that $supp(f) \cup supp(g) \subset supp(f \cup g)$. Let $x \in supp(f) \cup supp(g)$. Then $x \in supp(f)$ or $x \in supp(g)$. Thus $f(x) \neq 0$ or $g(x) \neq 0$. Hence $(f \cup g)(x) \neq 0$. It implies that $x \in supp(f \cup g)$. This completes the proof. □

Theorem 20. *Let f be a fuzzy subset of a ternary semiring T. Then f is a fuzzy alomost ternary subsemiring of T if and only if $supp(f)$ is an almost ternary subsemirng of T.*

Proof. Suppose that f is a fuzzy almost ternary subsemiring of T. We now have $(f+f)\cap f \neq 0$ and $[f\circ f\circ f]\cap f \neq 0$. Then there exists $x \in T$ such that $[(f\circ f\circ f)\cap f](x) \neq 0$. It follows that $f(x) \neq 0$ and $(f\circ f\circ f)(x) \neq 0$. Thus $x = abc$ for some $a, b, c \in T$ such that $f(a) \neq 0, f(b) \neq 0$, and $f(c) \neq 0$. Then $a, b, c \in supp(f)$. Hence $(C_{supp(f)} \circ C_{supp(f)} \circ C_{supp(f)})(x) \neq 0$ and $C_{supp(f)}(x) \neq 0$. It implies that $[(C_{supp(f)} \circ C_{supp(f)} \circ C_{supp(f)}) \cap C_{supp(f)})(x)] \neq 0$. Similarly, we obtain $[(C_{supp(f)} + C_{supp(f)}) \cap C_{supp(f)}](x) \neq 0$. Hence $C_{supp}(f)$ is a fuzzy almost ternary subsemiring of T. By Theorem 16, $supp(f)$ is an almost ternary subsemiring of T.

Conversely, suppose that $supp(f)$ is an almost ternary subsemiring of T. Then $C_{supp(f)}$ is a fuzzy almost ternary subsemiring of T. Thus $[(C_{supp(f)} + C_{supp(f)}) \cap C_{supp(f)}] \neq 0$ and $[(C_{supp(f)} \circ C_{supp(f)} \circ C_{supp(f)}) \cap C_{supp(f)}] \neq 0$. Then there exists $x \in T$ such that $[(C_{supp(f)} \circ C_{supp(f)} \circ C_{supp(f)}) \cap C_{supp(f)}](x) \neq 0$. Thus $(C_{supp(f)} \circ C_{supp(f)} \circ C_{supp(f)}) \neq 0$, and $C_{supp}(f)(x) \neq 0$. Hence there exists $a, b, c \in supp(f)$ such that $x = abc$. It follows that $f(a) \neq 0, f(b) \neq 0$, and $f(c) \neq 0$. Thus $[(f\circ f\circ f)\cap f](x) \neq 0$. This implies that $[(f\circ f\circ f)\cap f](x) \neq 0$. Thus $(f\circ f\circ f)\cap f \neq 0$. We can similarly show that $(f+f)(x) \neq 0$. Therefore f is a fuzzy almost ternary subsemiring of T. □

Proposition 21. *Let f, g be any fuzzy subsets of a ternary semiring T. If f is a fuzzy almost ternary subsemiring of T, then $supp(f \cup g)$ is a fuzzy almost ternary subsemiring of T.*

Proof. By Corollary 13, Proposition 19 and Theorem 20. □

4. Conclusions

In this paper, we have introduced almost ternary subsemirings of ternary semirings. We also studied the union and the intersection of two almost ternary subsemirings. We found that the union of almost ternary subsemirings of a ternary semiring is almost ternary subsemiring and the union of nonempty subsets of ternary semiring is an almost ternary subsemiring if there exists an almost ternary subsemiring in the union. Moreover, we presented their fuzzifications and provided the relationships between almost ternary subsemirings and their fuzzifications. It was shown that an almost subsemiring is equivelent to its characteristic mapping. Finally, we found that a fuzzy almost ternary subsemiring is equivalent to its support.

In the last of this section, there are some open problems that would be interesting for the readers to investigate as follows:

(1) We can study other properties of almost ternary subsemirings and fuzzy almost ternary subsemirings.

(2) We can extend the concept of almostity to define almost subalgebra of some algebraic structures, apply the concepts of fuzzy sets to define their fuzzications, and investigate the basic and advanced concepts of them.

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