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IMPACT OF THE NUSSELT AND RAYLEIGH NUMBERS ON HEAT TRANSFER IN RECTANGULAR ENCLOSURE

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Abstract: In this study, effect of the Nusselt and Rayleigh numbers on heat transfer in rectangular enclosure is investigated. The governing equations along with the boundary conditions are non-dimensionalized and discretized. The average Nusselt numbers ($\overline{Nu}$) for fluids of water ($Pr = 7$) and air ($Pr = 0.7$) at different Rayleigh numbers (Ra) are calculated. Details of empirical correlations used for calculating the average Nusselt numbers ($\overline{Nu}$) for specific fluids are presented. Mode of heat transfer in rectangular enclosure for suitable Nusselt and Rayleigh number is determined. Different factors which induce heat transfer in the rectangular enclosure are investigated. For each of these factors, appropriate numerical data is derived and elucidated. The numerical values of $\overline{Nu}$ are obtained as less than 1 at different Rayleigh numbers in $Ra < 1708$. It is determined that the average Nusselt numbers for water ($Pr = 7$) and air ($Pr = 0.7$) increases curved linearly. For different Rayleigh numbers in $Ra > 1708$, it is found that the average Nusselt numbers for water ($Pr = 7$) and air ($Pr = 0.7$) increases linearly. At $Ra = 1708$, the critical Rayleigh number, the fluid layer remains stable and hence quiescent. The transfer of heat from the top wall to the bottom wall within the enclosure is by conduction for liquid (water, $Pr = 7$) and through conduction and radiation for gas (air, $Pr = 0.7$).

AMS Subject Classification: 76D05
Key Words: average Nusselt number, Rayleigh number, heat transfer, rectangular enclosure, empirical correlations

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1. Introduction

Heat transfer in rectangular enclosures illustrates an internal flow. Internal flows are difficult to study but more significant than the external flows as illustrated by Bergman *et al.* [5]. It has wide range of applications in engineering and industry such as heating and cooling elements in electrical and nuclear industries. Hence, it is a significant area of research.

Analysis of combined convection, heat and mass transfer in a rectangular enclosure with wall heat and concentration sources was investigated by Ambethkar and Basumatary [1, 2, 3]. Alam *et al.* [4] have investigated natural convection in a rectangular enclosure due to partial heating and cooling at vertical walls. Effect of the temperature boundary conditions on free convective flow in rectangular enclosures was studied numerically by Corcione [6]. Heat and mass transfer of a conductive and convective flow with heat sources in a rectangular enclosure by using heat and streamlines was investigated by Deng and Tang [7]. Analyzing natural convection in rectangular enclosures with heat sources was investigated by Deng *et al.* [8]. Ghia *et al.* [9] have presented bench marked solutions for 2-D incompressible flow at high Reynold numbers in a cavity. Globe and Dropkin [10] have suggested empirical correlations for Nusselt numbers at different Rayleigh numbers of a natural convective heat transfer with heated walls. Hollands *et al.* [11] have proposed empirical correlations for average Nusselt number of free convective air and water flow in horizontal rectangular region. Heat transfer analysis in a porous fin enclosures was investigated by Hoseinzadeh [12]. Combined convective flow of nanofluids within different enclosures was investigated by Lzadi *et al.* [13]. Pellew and Southwell [14] have investigated convection in an enclosure whose walls are heated from below. Natural convection in a C- shaped enclosure filled with nanofluid was studied by Mohebbi *et al.* [16]. Takhar *et al.* [17] have investigated heat and mass transfer in a vertical moving cylinder. Motivation for investigating the heat transfer in rectangular enclosure is due to its significance as it illustrates internal flows which are complex and prominent to study than external flows. Exploring the effect of the Nusselt and Rayleigh numbers on heat transfer in a horizontal rectangular enclosure is an important task of research. Furthermore, the empirical correlation for average Nusselt numbers ($\overline{Nu}$) for different fluids are detailed. Based on the suggested numerical data, the modes of heat transfer within the rectangular enclosure are concluded.

This paper aims to investigate the impact of the average Nusselt numbers ($\overline{Nu}$) and the Rayleigh number (Ra) on heat transfer in a horizontal rectangular enclosure. Further, the mode of heat transfer due to the Nusselt and Rayleigh

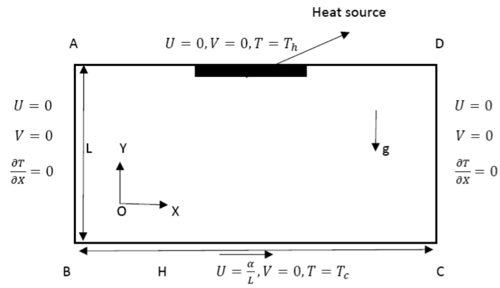

Figure 1: Schematic description

numbers between the walls of the enclosure is determined.

The summary of this paper comprises of mathematical formulation in Section 2 which includes the schematic, non-dimensional governing equations. Finite volume discretization is given in Section 3. Results and discussion are given in Section 4. conclusions are summarized under Section 5.

2. Mathematical formulation

2.1. Schematic description

A rectangular enclosure of height L and length H such that the top wall is assumed to be as a hot wall due to a heat source with temperature $T = T_h$ is placed on the entire top wall whereas the bottom wall is a cold wall with temperature $T = T_c$ as displayed in Figure 1 due to [3]. The left and the right wall is assumed to be insulated.

2.2. Mathematical equations

The non-dimensional form of the equations which govern the present problem are expressed due to [18] as follows:

continuity equation: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

x-momentum equation: $$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \quad (2)$$

y-momentum equation: $$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

$$+RaPr\theta, \quad (3)$$

energy equation: $$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{1}{RePr}\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right), \quad (4)$$

where x, y, L, u, v, P, θ, Pr, Re and Ra are dimensionless independent variables, height of the rectangle, dimensionless components of velocities along x and y-axis, dimensionless pressure, dimensionless temperature, the Prandtl number, the Reynolds number and the Rayleigh number which are defined as follows:

$$(x,y)=\frac{(X,Y)}{L},\quad (u,v)=\frac{(U,V)L}{\alpha},\quad P=\frac{p'L^2}{\rho\alpha^2},\quad \theta=\frac{T-T_c}{T_h-T_c},$$
$$Pr=\frac{\nu}{\alpha}, Re=\frac{u_0L}{\nu},\quad Ra=\frac{g\beta_T(T_h-T_c)}{\alpha\nu}L^3.$$

The boundary conditions in dimensionless form becomes as:

$$\left.\begin{array}{llll} \text{on AB:} & \text{at } x=0, & u=v=0, & \dfrac{\partial\theta}{\partial x}=0,\\ \text{on DC:} & \text{at } x=2, & u=v=0, & \dfrac{\partial\theta}{\partial x}=0,\\ \text{on BC:} & \text{at } y=0, & u=1, v=0, & \theta=0,\\ \text{on AD:} & \text{at } y=1, & u=v=0, & \theta=1, \end{array}\right\} \quad (5)$$

where AB, DC, BC and AD are the left, right, downside and upside walls of the four-sided enclosure.

3. Method of solution

3.1. Control Volume Discretization

The method of solution is by using the upwind control volume method ([18], pp.197–200). The crux of the control volume method lies in computing the convective flux and diffusive conductance at the cell faces of each of the control volume as given in ([18], pp.197–200) on this subject.

The equation of continuity is discretized at the node (I,J) which reduces to

$$F_e-F_w+F_n-F_s=0. \quad (6)$$

The discretized x-momentum equation at (i,J) reduces to

$$a_{i,J}u_{i,J}=\sum a_{nb}u_{nb}+(P_{I-1,J}-P_{I,J})A_{i,J}, \quad (7)$$

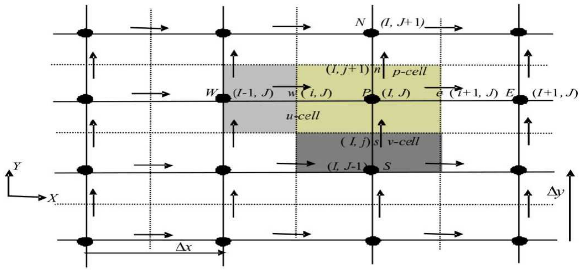

Figure 2: Staggered grid

where $A_{i,J}$ is area and neighbors of $\sum a_{nb}u_{nb}$ will be $(i+1, J)$, $(i-1, J)$, $(i, J+1)$ and $(i, J-1)$. The coefficients of upwind difference scheme along with fluxes and conductance at cell faces of the u-control volume are computed as given in ([18], pp.197–199):

$$a_{i,J} = a_{i-1,J} + a_{i+1,J} + a_{i,J+1} + a_{i,J-1} + \Delta F. \tag{8}$$

The discretized y-momentum equation at the node (I, j) reduces to

$$a_{I,j}v_{I,j} = \sum a_{nb}v_{nb} + (P_{I,J-1} - P_{I,J})A_{I,j} + b_{I,j}. \tag{9}$$

For v-control volume, the values of F and D, can be determined in a similar way as suggested in Patankar ([15], p.122) for each of the faces e, w, n and s.

Pressure correction equation as per ([18], p.202) reduces to

$$a_{I,J}P'_{I,J} = a_{I+1,J}P'_{I+1,J} + a_{I-1,J}P'_{I-1,J} + a_{I,J+1}P'_{I,J+1} + a_{I,J-1}P'_{I,J-1} + b'_{I,J}. \tag{10}$$

Discretizing the energy equation at the node (I,J) reduces to

$$a_{I,J}\theta_{I,J} = \sum a_{nb}\theta_{nb}. \tag{11}$$

3.2. Numerical computations

An algorithm for calculating the solutions of the flow variables is given below:

Algorithm

A modified algorithm based on ([18], p.202) consists of the following steps:

1. Guess all flow variables.

2. Solve the equation of momentum for (u^*, v^*).

3. Solve the equation of pressure-correction for $p^{'}$.

4. Evaluate P as a sum of $P^{'}$ and P^*. Evaluate both velocities (u, v) by using the guessed velocities (u^*, v^*) and corrected pressure $P^{'}$.

5. Solve the temperature discretized equations.

6. Replace $(p^*, u^*, v^*, \theta^*)$ with corrected values (p, u, v, θ) return to step 2 and repeat the process until the solution converges.

4. Results and discussion

Different factors which influence the heat transfer in rectangular enclosures are the angle of tilt, the aspect ratio and the non-dimensional parameters such as the Nusselt number Nu and the Rayleigh number Ra and the Prandtl number Pr. The Nusselt number is the ratio of convection to the pure conduction heat transfer. The Rayleigh number is the product of the Grashof number Gr and the Prandtl number. The Grashof number is the ratio of the buoyancy forces to the viscous forces. When the angle of tilt is 180°, then the upper wall is heated and the lower wall is cooled for a rectangular enclosure. When $Ra < 1700$, the heat transfer in the rectangular enclosure is by conduction. Different modes of heat transfer within the rectangular enclosure are by conduction, convection and the radiation. This study concludes exact mode of the heat transfer within the enclosure in different cases.

The average Nusselt number $(\overline{Nu})$ represents the overall convective flow within the rectangular enclosure. In order to determine the total heat transfer from the upper wall of the rectangular region as shown in Figure 1, we need to determine the average Nusselt number $\overline{Nu}$ which is defined as $\overline{Nu}=\frac{\bar{h}L}{k}$, where $\bar{h}$ is the average convection coefficient and k is the thermal conduction. The average Nusselt number represents the overall convection heat transfer occurring at the entire top wall of the rectangular enclosure. The average Nusselt numbers $\overline{Nu}$ for fluids (water and air) are calculated by using $\overline{Nu} = 0.069(Ra_L)^{\frac{1}{3}} Pr^{0.074}$, the empirical correlations for heat transfer in horizontal rectangular enclosures as proposed by Globe and Dropkin [8]. Hence, the goal of this study is achieved by calculating the average Nusselt numbers $(\overline{Nu})$ for fluids by using this empirical correlation and are tabulated in Tables 1 and 2 which are given below. The Rayleigh numbers (Ra) applicable to the problem of horizontal rectangular enclosure heated on the top wall are $Ra \leq 1708$. The

Table 1: Average Nusselt numbers ($\overline{Nu}$) for $Pr = 7$ at different $Ra < 1708$

Type of fluid Liquid	Rayleigh number(Ra)	Average Nusselt numbers ($\overline{Nu}$)
water($Pr = 7$)	1	0.07968
	10	0.17168
	100	0.369873
	1000	0.796868
	1700	0.951046
	1707	0.952352

Table 2: Average Nusselt numbers ($\overline{Nu}$) for $Pr = 0.7$ at different $Ra < 1708$

Type of fluid Gas	Rayleigh number(Ra)	Average Nusselt numbers ($\overline{Nu}$)
air($Pr = 0.7$)	1	0.067202
	10	0.144784
	100	0.311927
	1000	0.672026
	1700	0.802052
	1707	0.803152

average Nusselt numbers ($\overline{Nu}$) for fluids (water and air) at different Rayleigh numbers of 1, 10, 100, 1000, 1700, 1707 in the range of $Ra < 1708$ are tabulated in Tables 1 and 2 and are sketched in Figures 3 and 4. It is found that the values of $\overline{Nu}$ are less than 1 at these Rayleigh numbers. It is observed from Figures 3 and 4 that, the average Nusselt numbers for water ($Pr = 7$) and air ($Pr = 0.7$) increases curved linearly. Hence, the transfer of heat within the enclosure is by conduction for liquid (water, $Pr = 7$) and for gas (air, $Pr = 0.7$) through conduction and radiation.

The average Nusselt numbers ($\overline{Nu}$) for fluids at different Rayleigh numbers of 1709, 1750, 1800, 2000, 3000, 3295 in the range of $Ra > 1708$ are tabulated in 3 and 4 and are sketched in figures 5 and 6. It is observed from figures 5

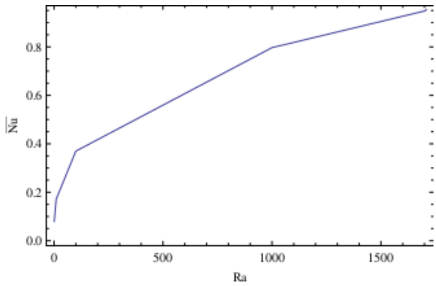

Figure 3: $\overline{Nu}$ for water ($Pr = 7$) at $Ra < 1708$.

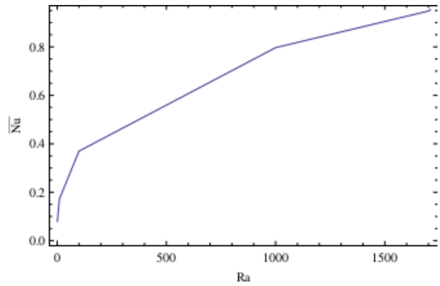

Figure 4: $\overline{Nu}$ for air ($Pr = 0.7$) at $Ra < 1708$.

Table 3: Average Nusselt numbers ($\overline{Nu}$) for $Pr = 7$ at different $Ra > 1708$

Type of fluid Liquid	Rayleigh number(Ra)	Average Nusselt numbers ($\overline{Nu}$)
water($Pr = 7$)	1709	0.952724
	1750	0.960283
	1800	0.969342
	2000	1.00399
	3000	1.14928
	3295	1.18578

and 6 that, the average Nusselt numbers for water ($Pr = 7$) and air ($Pr = 0.7$) increases linearly. When $Ra = 1708$, at the critical Rayleigh number Pellew and Southwell [12], the fluid layer remains stable and hence quiescent. There is a decrease in heat transfer from the hot to the cold wall. So, for all $Ra \leq 1708$, there is no advection within the enclosure.

5. Conclusions

In this investigation, the average Nusselt numbers $\overline{Nu}$ for fluids of water ($Pr = 7$) and air ($Pr = 0.7$) at different Rayleigh numbers in the range of $Ra < 1708$ are calculated. These calculations are done at different Rayleigh numbers of 1, 10, 100, 1000, 1700, 1707 by using the empirical correlations. It is found that the values of $\overline{Nu}$ are less than 1 at these Rayleigh numbers. It is determined

Table 4: Average Nusselt numbers ($\overline{Nu}$) for $Pr = 0.7$ at different $Ra > 1708$

Type of fluid Gas	Rayleigh number(Ra)	Average Nusselt numbers ($\overline{Nu}$)
air($Pr = 0.7$)	1709	0.803465
	1750	0.80984
	1800	0.81748
	2000	0.8467
	3000	0.96923
	3295	1.0001

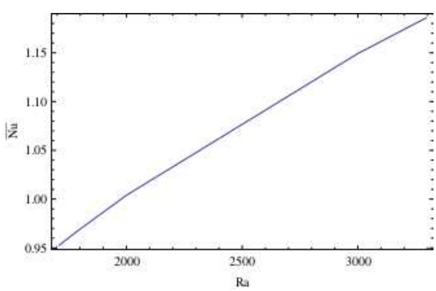

Figure 5: $\overline{Nu}$ for water ($Pr = 7$) at $Ra > 1708$.

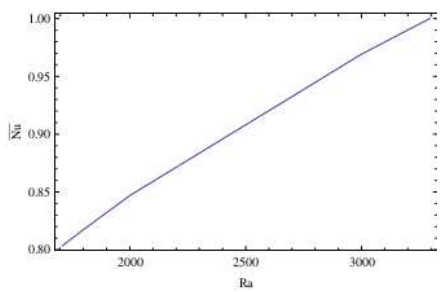

Figure 6: $\overline{Nu}$ for air ($Pr = 0.7$) at $Ra > 1708$.

that the average Nusselt numbers for water ($Pr = 7$) and air ($Pr = 0.7$) increases curved linearly. Hence, the transfer of heat from the top wall to the bottom wall within the enclosure is by conduction for liquid (water, $Pr = 7$) and through conduction and radiation for gas (air, $Pr = 0.7$). Therefore, there is no advection in the enclosure. The average Nusselt numbers ($\overline{Nu}$) for fluids at different Rayleigh numbers of 1709, 1750, 1800, 2000, 3000, 3295 in the range of $Ra > 1708$ are tabulated and sketched. It is found from these tabulations that the average Nusselt numbers for water ($Pr = 7$) and air ($Pr = 0.7$) increases linearly. At $Ra = 1708$, the critical Rayleigh number [11], the fluid layer remains stable and hence quiescent. There is a decrease in heat transfer from the hot to the cold wall.

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