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ON THE STABILYTY OF A MAGNETOELASTIC SYSTEM IN THE PRESENCE OF SMALL FRICTION

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Abstract: The paper considers the influence of small nonlinear friction on the occurrence of instability in a dynamic system consisting of two conductive thin plates in a magnetic field.

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1. Introduction

In papers [1] – [5], we investigated some problems of the stability of oscillations of magnetoelastic objects - conducting plates and shells in the presence of a magnetic field. Particular attention was paid to the study of the oscillation regime in the area of instability. It was shown that in such dynamic systems with a diamagnetic gap, at certain speeds of motion of the interfaces, an area of instability arises, as a result of which the amplitude of the oscillations increases.

In this paper, we study the effect of small nonlinear friction on the result of oscillations in such a system. Friction is given by a small parameter, and thus we have a problem *with a small parameter*. For its study, we use techniques and methods dating back to the works of Poincaré, Van Der – Pol [6], N. M. Krylov, N. N. Bogolyubov and Yu. A. Mitropolsky [7] – [9]. First of all, here

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we are talking about the asymptotic method [8] and the method of averaging [7], [9].

2. Preliminary Notes

Let us consider a system formed by two thin ideally conducting plates of thickness h, with bending D rigidity and material density ρ, separated by a vacuum diamagnetic gap filled with a constant and uniform magnetic field directed along the axis x. Let the plate $z=0$ move along the axis x with the speed v_0, and the plate $z=l$ is stationary.

The system of equations describing oscillations in the plates has the following form [4]

$$\rho h\left(\frac{\partial}{\partial t}+v_0\frac{\partial}{\partial x}\right)^2\xi_1=-D\frac{\partial^4\xi_1}{\partial x^4}-P_{1m}, \tag{1}$$
$$\rho h\frac{\partial^2\xi_2}{\partial t^2}=-D\frac{\partial^4\xi_2}{\partial x^4}+P_{2m},$$

where $\xi\left(x,t\right)$ - small vertical displacements of the points of the plate surface are proportional to $\exp\left(i\left(\omega t-kx\right)\right)$, P_{1m} and P_{2m}- magnetic pressures on the surface of the plates.

With transverse deformations in the plates, friction occurs equal to

$$F_{f\,r}=\varepsilon f\left(\frac{\partial\xi}{\partial t}\right),\ \ \varepsilon\ll 1. \tag{2}$$

To take this force into account, we will add the friction force (2) to the right side of each equation of system (1). Then we get

$$\rho h\left(\frac{\partial}{\partial t}+v_0\frac{\partial}{\partial x}\right)^2\xi_1=-D\frac{\partial^4\xi_1}{\partial x^4}-P_{1m}+\varepsilon f\left(\frac{\partial\xi_1}{\partial t}\right), \tag{3}$$
$$\rho h\frac{\partial^2\xi_2}{\partial t^2}=-D\frac{\partial^4\xi_2}{\partial x^4}+P_{2m}+\varepsilon f\left(\frac{\partial\xi_2}{\partial t}\right).$$

For convenience, we express the magnetic pressures on the surface of the plates in terms of the potential Ψ :

$$P_{1m}=\frac{H_0}{4\pi}\cdot\frac{\partial\Psi}{\partial x}\quad at\quad z=0,$$

$$P_{2m}=\frac{H_0}{4\pi}\cdot\frac{\partial\Psi}{\partial x}\quad at\quad z=l.$$

Then system (3) takes the form

$$\rho h\left(\frac{\partial}{\partial t}+v_0\frac{\partial}{\partial x}\right)^2\xi_1=-D\frac{\partial^4\xi_1}{\partial x^4}+\varepsilon f\left(\frac{\partial\xi_1}{\partial t}\right)-\frac{H_0}{4\pi}\cdot\frac{\partial\Psi}{\partial x}, \tag{4}$$

$$\rho h\frac{\partial^2\xi_2}{\partial t^2}=-D\frac{\partial^4\xi_2}{\partial x^4}+\varepsilon f\left(\frac{\partial\xi_2}{\partial t}\right)+\frac{H_0}{4\pi}\cdot\frac{\partial\Psi}{\partial x}.$$

The potential Ψ in this case satisfies the equation

$$\frac{\partial^2\Psi}{\partial t^2}+\frac{\partial^2\Psi}{\partial x^2}=0 \tag{5}$$

with boundary conditions

$$\frac{\partial\Psi}{\partial z}=H_0\frac{\partial\xi_1}{\partial x}\quad at\quad z=0,$$

$$\frac{\partial\Psi}{\partial z}=H_0\frac{\partial\xi_2}{\partial x}\quad at\quad z=l.$$

If there is no friction, that is $\varepsilon=0$, then the system of equations (4), (5) admits a solution in the form

$$\xi_1=a_1\cos\theta_1,\quad \xi_2=a_2\cos\theta_2, \tag{6}$$

$$\frac{\partial\xi_1}{\partial t}=-a_1\omega\sin\theta_1,\quad \frac{\partial\xi_2}{\partial t}=-a_2\omega\sin\theta_2,$$

where

$$\theta_1=\omega\,t-k\,x+\varphi_1,\ \theta_2=\omega\,t-k\,x+\varphi_2\ . \tag{7}$$

Natural frequencies of oscillations in the absence of friction are determined by the expression [4]

$$\omega=\frac{kv_0}{2}\pm\sqrt{\Omega^2+\left(\frac{kv_0}{2}\right)^2\pm\sqrt{\Omega_0^4+k^2v_0^2\Omega^2}}.$$

The condition under which one of the roots becomes complex is determined by the inequalities [4]

$$2v_{1\Phi}<v_0<2v_{2\Phi}. \tag{8}$$

In doing so

$$\omega=\omega_0\pm i\delta,\quad \omega_0=\frac{1}{2}\,kv_0.$$

3. Oscillation of plates in the presence of small non-linear friction

Let now in the system of equations (4) $\varepsilon \neq 0$. Since we have $\varepsilon > 0$, $\varepsilon \ll 1$, we can consider ε it a **small parameter**. Consequently, our problem becomes a dynamic problem of studying oscillations with a small parameter. We use the methodology proposed by N. N. Bogolyubov and N. M. Krylov [7]. We will assume that a_1, a_2 from (6) and φ_1, φ_2 from (7) are new functions that are to be determined. Performing the change of variables in the form (6), we obtain the following system of equations for these new unknowns

$$\cos\theta_1 \cdot \frac{da_1}{dt} - a_1 \sin\theta_1 \cdot \frac{d\varphi_1}{dt} = 0,$$
$$\rho h \left(-\omega\, \sin\theta_1 \cdot \frac{da_1}{dt} - \omega\, a_1 \cos\theta_1 \cdot \frac{d\varphi_1}{dt} \right) = \varepsilon f\left(\frac{d\xi_1}{dt} \right),$$
$$\cos\theta_2 \cdot \frac{da_2}{dt} - a_2 \sin\theta_2 \cdot \frac{d\varphi_2}{dt} = 0,$$
$$\rho h \left(-\omega\, \sin\theta_2 \cdot \frac{da_2}{dt} - \omega\, a_2 \cos\theta_2 \cdot \frac{d\varphi_2}{dt} \right) = \varepsilon f\left(\frac{d\xi_2}{dt} \right).$$

We slightly transform this system of equations by introducing a function

$$\hat{f}\left(-\omega\, a_i \sin\theta_i\right) = \frac{1}{\rho h} f\left(-\omega\, a_i \sin\theta_i\right) = \frac{1}{\rho h} f\left(\frac{\partial \xi_i}{\partial t} \right), \quad i = 1, 2.$$

We get

$$\begin{gathered}
\cos\theta_1 \cdot \frac{da_1}{dt} - a_1 \sin\theta_1 \cdot \frac{d\varphi_1}{dt} = 0, \\
-\omega\, \sin\theta_1 \cdot \frac{da_1}{dt} - \omega\, a_1 \cos\theta_1 \cdot \frac{d\varphi_1}{dt} = \varepsilon \hat{f}\left(-\omega\, a_1 \sin\theta_1\right), \\
\cos\theta_2 \cdot \frac{da_2}{dt} - a_2 \sin\theta_2 \cdot \frac{d\varphi_2}{dt} = 0, \\
-\omega\, \sin\theta_2 \cdot \frac{da_2}{dt} - \omega\, a_2 \cos\theta_2 \cdot \frac{d\varphi_2}{dt} = \varepsilon \hat{f}\left(-\omega\, a_2 \sin\theta_2\right).
\end{gathered} \tag{9}$$

Solving this system of equations, we find $\frac{da_i}{dt}$ and $\frac{d\varphi_i}{dt}$:

$$\frac{da_i}{dt} = -\frac{\varepsilon}{\omega} \hat{f}\left(-\omega\, a_i \sin\theta_i\right) \sin\theta_i,$$

$$\frac{d\varphi_i}{dt} = -\frac{\varepsilon}{\omega\, a_i} \hat{f}\left(-\omega\, a_i \sin\theta_i\right) \cos\theta_i,$$

where $i = 1, 2$.

To study the effect of friction on the oscillations of our system, it is necessary to specify the friction.

Let the friction in the plates be given as [6], [8]

$$f\left(\frac{\partial \xi}{\partial t}\right) = -\gamma \frac{\partial \xi}{\partial t}\left(1 + \mu \left(\frac{\partial \xi}{\partial t}\right)^2\right).$$

Using the averaging procedure [9], we obtain, in the first approximation, the following system of equations for the functions.

Using the averaging procedure [9], we obtain, in the first approximation, the following system of equations for the functions a and φ:

$$\frac{da}{dt} = \frac{1}{2}\varepsilon \frac{\gamma}{\rho h} a \left(1 + \frac{3}{4}\mu \omega^2 a^2\right),$$

$$\frac{d\varphi}{dt} = 0.$$

The solution to this system of equations has the form

$$a = \frac{a_0 \exp\left(-\frac{1}{2}\varepsilon \frac{\gamma t}{\rho h}\right)}{\sqrt{1 + \frac{3}{4}\mu\, \omega^2 a_0^2 \left(1 - \exp\left(-\varepsilon \frac{\gamma t}{\rho h}\right)\right)}}, \tag{10}$$

$$\varphi = const.$$

Thus, in the first approximation, the perturbation of the plates has the form of a traveling wave, so that

$$\xi = \mathrm{Re} \frac{a_0 \exp\left(-\frac{1}{2}\varepsilon \frac{\gamma t}{\rho h}\right) \cdot \cos\left(\omega t - kx + \varphi\right)}{\sqrt{1 + \frac{3}{4}\mu \omega^2 a_0^2 \left(1 - \exp\left(-\varepsilon \frac{\gamma t}{\rho h}\right)\right)}}. \tag{11}$$

The frequency of oscillations in a wave in this approximation is equal to the natural frequency of oscillations of the plate in the absence of friction, and the amplitude decreases with time according to the law (10). If the speed of the relative motion of the plates is in the interval determined by inequality (8), then the frequency becomes complex. Let $\delta = \mathrm{Im}\omega$, $\delta \ll \omega_0 = \frac{kv_0}{2}$. Taking this condition into account, solution (11) can be written in the form

$$\xi = \frac{a_0 \exp\left(-\frac{1}{2}\varepsilon \frac{\gamma t}{\rho h}\right) \cdot ch\delta t \cdot \cos\left(\omega_0 t - kx + \varphi\right)}{\sqrt{1 + \frac{3}{4}\mu \omega_0^2 a_0^2 \left(1 - \exp\left(-\varepsilon \frac{\gamma t}{\rho h}\right)\right)}}.$$

After analyzing this expression, we see that the regime of oscillation of the plates depends on the relationship between the parameters δ and $\frac{\varepsilon\gamma}{2\rho h}$. If $\delta < \frac{\varepsilon\gamma}{2\rho h}$, then the oscillations of the plates will be damped. If $\delta > \frac{\varepsilon\gamma}{2\rho h}$, then the oscillations will be increasing. If $\delta = \frac{\varepsilon\gamma}{2\rho h}$, then there will be a stationary periodic regime of oscillations

$$\xi = \frac{a_0 \cdot \cos(\omega_0 t - kx + \varphi)}{2\sqrt{1 + \frac{3}{4}\mu\omega_0^2 a_0^2}} = A\cos(\omega_0 t - kx + \varphi). \tag{12}$$

The power dissipated in the system due to friction is determined in the form

$$P = F_{fr}\frac{\partial \xi}{\partial t} = \gamma\left(\frac{\partial \xi}{\partial t}\right)^2\left(1 + \mu\left(\frac{\partial \xi}{\partial t}\right)^2\right).$$

In the case of stationary periodic oscillations (12), the power in the system is equal to In the case of stationary periodic oscillations (12), the power in the system is equal to

$$P = \gamma A^2\omega_0^2 \sin^2(\omega_0 t + \varphi)\left(1 + \mu A^2\omega_0^2 \sin^2(\omega_0 t + \varphi)\right).$$

Averaging this expression over time, we obtain that the power dissipated in the system, averaged over the period of oscillations, is equal to

$$P = \frac{1}{2}\gamma A^2\omega_0^2\left(1 + \frac{3}{4}\mu A^2\omega_0^2\right). \tag{13}$$

Thus, the vacuum diamagnetic gap does not eliminate the dissipation of the energy of the system and, consequently, friction during sliding, since the calculated average power (13) can be considered as the power corresponding to some effective friction force acting when the surfaces move.

4. Conclusion

The influence of small nonlinear friction in thin plates on the stability of a rupture formed by a vacuum diamagnetic gap has been studied. It is shown that the presence of small friction leads, under certain conditions, to the existence of a stationary regime of oscillations in the area of instability. The average power dissipating in the plates is determined, which corresponds to the effective friction force acting when the plates are sliding, separated by a vacuum gap.

Thus, the presence of a diamagnetic gap leads to a specific instability, the onset of which has the same form for all dynamic models considered by us.

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